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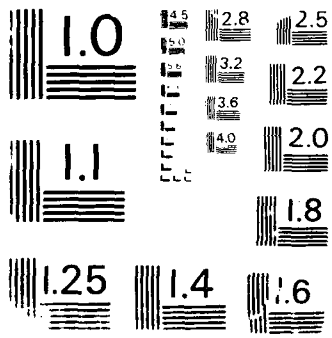
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This report is a part of a Corps-wide project to provide guidance for the use of finite element analyses. A continuation of the Phase Ia study dealing with a static finite element analysis of a gravity dam and Phase Ib study dealing with foundation-structure interaction, Phase II gives direction in performing a response-spectrum dynamic finite element analysis of a gravity dam. An example problem is presented to demonstrate how to verify the results for a simplified structure before moving to the analysis of the real structure. An analysis of a realistic gravity dam is then presented with results.

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PREFACE

This report is aimed at providing guidance for the use of the finite element method of analysis for the analysis of concrete gravity dams. This Phase II report will address only the dynamic analysis of the gravity dam and the need for including a foundation. The Phase Ia report addressed the static analysis of the gravity dam. Phase Ib will address the effect of the foundation in the static analysis of concrete gravity dams. Other reports will address guidance for other phases of finite element analysis. The work was sponsored under funds provided to the US Army Engineer Waterways Experiment Station (WES) by the Civil Works Directorate, Office, Chief of Engineers (OCE), US Army, as part of the Computer-Aided Structural Engineering (CASE) Project.

Input for the report was obtained from the CASE Task Group on Finite Element Analysis. Members and others who directly contributed to the report were:

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COL Larry B. Fulton, EN, is the Commander and Director of WES. Dr. Robert W. Whalin is the Technical Director.

CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
inches	2.54	centimetres
kips per foot	1355.818	newton-metres
kips per square foot	47.88026	kilopascals
pounds	4.448222	newtons
pounds per cubic foot	16.01846	kilograms per cubic metre
pounds per foot	14.5939	newtons per metre
pounds per square foot	47.88026	pascals
pounds per square inch	6.894757	kilopascals
slugs	14.5939	kilograms

THE RESPONSE-SPECTRUM DYNAMIC ANALYSIS OF GRAVITY DAMS USING THE FINITE ELEMENT METHOD

Phase II

PART I: INTRODUCTION

Background

1. This report has been prepared as part of the ongoing effort by the Computer-Aided Structural Engineering (CASE) Committee on finite element analysis. It is part of a Corps-wide project to provide guidance for the use of finite element analysis.

Objective

2. The primary objective of this study is to give direction in performing a response-spectrum dynamic finite element analysis of a gravity dam using a general-purpose computer program. This is one of several methods currently being used within the Corps of Engineers (CE) for performing a dynamic analysis on gravity dams based on guidance provided in ETL 1110-2-303 (Department of the Army 1985). Other procedures and computer programs commonly used are outlined below with the differences between them discussed. Table 1 contains a summary of the procedures and computer programs.

a. The seismic coefficient method as presented in EM 1110-2-2200 (Department of the Army 1958) is in reality a static analysis with static forces representing inertial and hydrodynamic loads. The dam is analyzed using elementary beam theory and assumes a fixed foundation and incompressible reservoir. The hydrodynamic pressure distribution increases with depth. This procedure is good for horizontal motions only. The procedure is currently used only to evaluate overturning and sliding stability.

b. The "simplified" method for earthquake analysis of gravity dams was developed by A. K. Chopra (1978). The analysis of the dam is similar to that of the seismic coefficient method in that the inertial and hydrodynamic loads are represented as static forces and the dam is analyzed using elementary beam theory. The simplified method also assumed a rigid foundation. The difference between the seismic coefficient method and the 1978 version of the simplified method lies in the way that the inertial and hydrodynamic loads are determined. In the simplified method these loads are based on the fundamental mode of vibration of

Table 1

Currently Used Procedures Within the Corps of Engineers for Dynamic Analysis of Gravity Dams

Procedure	Type of Analysis	Dam Model	Reservoir Model	Foundation Model	Earthquake Motion
Seismic coefficient method	Static	Rigid (elementary beam theory)	Incompressible (applied to dam as static loads)	Fixed	Horizontal only
Simplified method (1978 version)	Static (with hydrodynamic loads determined using response-spectrum method)	Rigid (elementary beam theory)	Compressible (applied to dam as static loads)	Fixed	Horizontal only
Simplified method (1986 version)	Static (with hydrodynamic loads determined using response-spectrum method)	Rigid (elementary beam theory)	Compressible with effects of sedimentation added (applied to dam as static loads)	Modelled using a period lengthening ratio and added damping	Horizontal only
Cole/Cheek program	Static (with hydrodynamic loads determined using response-spectrum method)	FEM (linear elastic)	Compressible (applied to dam as static loads)	Fixed	Horizontal only
EADHI program	Time-history (frequency domain)	FEM (linear elastic)	Compressible (added mass)	Fixed	Horizontal and vertical
EAGD-84 program	Time-history (frequency domain)	FEM (linear elastic)	Compressible (added mass)	Viscoelastic halfplane	Horizontal and vertical
General purpose 2-D and 3-D FEM programs	Time-history or response-spectrum	FEM (linear elastic)	Incompressible (added mass, typ. Westergaard)	Fixed or FEM	Horizontal and vertical

the dam, with and without reservoir. In this method the reservoir is assumed to be compressible. As with the seismic coefficient method, only horizontal motions are considered. In 1986, Fenves and Chopra revised the simplified method to include the effects of an elastic foundation (by using a period lengthening ratio and added damping) and the effects of sediments in the reservoir.

c. In 1986, Cole and Cheek (Technical Report SL-86-44, Department of the Army) combined the simplified method (1978 version) with a finite element model of the dam (in lieu of using elementary beam theory). At present the procedure assumes a rigid foundation but plans are to include the effects of a linear elastic foundation modeled by a finite element grid.

d. EADHI (a computer program for Earthquake Analysis of Gravity Dams Including Hydrodynamic Interaction) is a finite element program that uses a time-history (but in the frequency domain) method of dynamic analysis. The program assumes a rigid foundation and models the reservoir as a compressible fluid. The hydrodynamic effect is represented by applying added mass to the face of the dam. The added mass is computed using methods similar to those used in the simplified method discussed earlier.

e. The EAGD-84 (Earthquake Analysis of Gravity Dams Including Hydrodynamic and Foundation Interaction) program is very similar to the EADHI program except that the foundation is modeled as a viscoelastic halfplane.

f. Several general-purpose programs are available that will perform either a 2-D or 3-D finite element time-history or response-spectrum dynamic analysis. Both dam and foundation can be modeled by a linear elastic finite element grid or the dam can be modeled on a fixed foundation. The influence of the reservoir on the dam is typically modeled using the concept of added mass. Westergaard's added mass method is commonly used which assumes a rigid dam and incompressible fluid.

3. This study is a continuation of the Phase Ia study which familiarized the beginning finite element analyst with the steps necessary to perform a static finite element analysis of a gravity dam (Will 1987). Many of the steps in performing this dynamic analysis could also apply to other Corps structures. The beginning engineer should be able to develop an understanding of the steps necessary to perform a dynamic analysis by carefully following this study and using supplemental material.

4. Following is a list of the necessary steps in performing a dynamic finite element analysis. The steps are very similar to those for a static analysis as presented in the Phase Ia report (Will 1987).

a. Select a finite element computer program currently in use by the Corps or in widespread use by private engineering firms and supported by a vendor with the desired analysis capabilities.

b. Select a simple problem as close as possible in overall geometry, material properties, boundary conditions, and loading conditions as the real structure to be analyzed. This structure should have closed-form, experimental, or other analytical solution results available.

c. Select the type of analysis (i.e. response-spectrum, time-history) to be performed to obtain the desired results.

d. Select the finite element types to be used in the analysis from the library of elements available in the program chosen in Step a.

e. Develop and analyze finite element models of the simplified structure and compare results, such as deflections and stresses, with the closed-form results.

f. Develop modeling guidelines for both the grid and loadings from the results of Step e, which may be extended to the real structure.

g. Prepare a finite element model of the real structure and perform an analysis.

h. Ask the following question: Is the solution accurate? If the answer is no, refine and reanalyze until the answer is yes.

5. Before actually performing the analysis, further detailed discussion of these steps is warranted to understand their necessity:

a. In Step a, the key concept is that the finite element program should be currently used by the Corps or other engineering firms and supported by a vendor. There are numerous finite element programs available today, therefore care must be taken in the selection process. While factors such as ease of use, functional capabilities, and price are extremely important, an overriding consideration is the use of the program within the Corps or other engineering firms and support by the vendor. An ideal situation is to find a program that is easy to use, has the necessary functional capabilities, is reasonably priced, and is currently being used by someone within the engineer's group and is supported by the vendor.

b. The motivation for Steps a-e is to provide an opportunity for the engineer to build confidence in the use of the program, finite element modeling techniques, and to develop an understanding of the convergence criteria. Another important reason for these steps is to provide the engineer with an understanding of the type, quantity, and quality of finite element results. A much too common occurrence is for the engineer to devote an enormous amount of time in developing the finite element model and after results have been obtained, too little time is devoted to the interpretation of these results, i.e., the accuracy of the results, or their actual usage in the design process.

c. From the analysis performed in Steps b-e, the engineer must then extrapolate the information gained from the modeling of the simple structure to the modeling of the real structure. Guidelines such as the number of subdivisions of the mesh in the horizontal and vertical directions may be developed for use in the first model of the real structure. Extreme care should also be used when deciding how to handle the mass distribution of the structure and the necessity to include any additional mass, such as foundation and adjacent water.

d. In Step g, the real structure is modeled, analyzed, and the results are interpreted. This leads to the crucial questions in the analysis: Is the solution accurate within an error criteria developed by the engineer? How much error is there? These are the most difficult and crucial questions in the entire process. In many instances, the only correct way to answer these questions is to refine the model, reanalyze, and compare solutions. The following question should then be asked: Have the results changed significantly due to the refinement? If not, an approximate solution has converged and the engineer must determine if the results make physical sense. If the results have changed significantly, other models may be required and comparisons repeated until convergence is satisfied. The engineer must remember that the finite element method is an approximate solution technique.

e. Also the question of whether to perform a time-history or response-spectrum analysis should be considered. In either case the dynamic loadings and mass distribution should be examined to ensure that they are appropriate.

6. In performing these steps for the analysis of a gravity dam, this phase of the study is limited to developing a method to analyze the deflections and stresses of the gravity concrete structure only. Interaction between the structure and foundation is not considered at this time, however the reservoir is considered. The program selected in Step a was GTSTRUDL* since it is supported by the vendor, and currently widely used by the Corps. Also, GTSTRUDL is representative of a general-purpose finite element program.

7. PART II of this report presents an example of Steps b-e in preparation for the analysis of a gravity dam. An actual analysis of a nonoverflow monolith similar to the Richard B. Russell Dam is presented in PART III. The same example problem is then analyzed using Chopra's simplified method to illustrate the different results produced by each method. Part IV studies the impace of including a foundation with regard to combined static-dynamic stresses within a gravity dam.

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* GTSTRUDL is a general-purpose finite element program owned and maintained by the GTICES Systems Laboratory, School of Civil Engineering, Georgia Institute of Technology. Program runs used in this report were made on the Control Data Corporation, Cybernet Computer System.

PART II: FINITE ELEMENT DYNAMIC ANALYSIS OF A SIMPLIFIED STRUCTURE

Selection of a Simplified Structure

8. The simplified cantilever structure representative of an idealized gravity dam (Figure 1), finite element models, parameters, along with

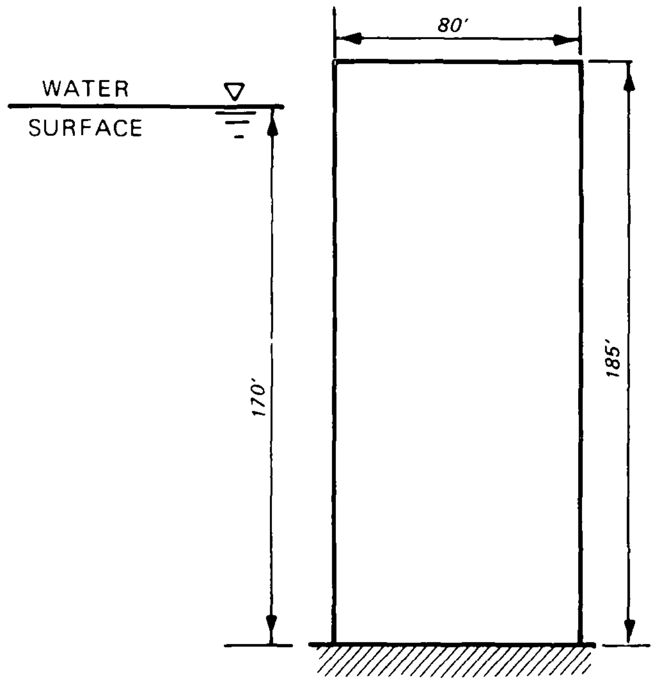

Figure 1. Simplified structure

recommendations established in Phase Ia of this study were used here. Again, the finite element runs were all made using GTSTRUDL. The program can be used in the analysis of the static and dynamic response of linear two- and three-dimensional (2- and 3-D) structural systems. The element used was the "IPQQ" eight node isoparametric quadratic quadrilateral element.

Finite Element Models

Finite element meshes

9. Three different models previously developed in the Phase Ia report to compare the convergence characteristics were used again. The various models are called the coarse, fine, and very fine meshes to indicate the relative degree of refinement. They are also referred to as Meshes 1, 2, and 3 as

shown below. These meshes are illustrated in Figures 2, 3, and 4. The node and elements are labeled in these figures. A summary of the meshes is presented below:

	Description	Number of Nodes	Number of Elements (All IPQQ's)
Mesh 1	Coarse	45	10
Mesh 2	Fine	149	40
Mesh 3	Very fine	537	160

Modeling procedure

10. The models were assumed to be completely restrained along their bases and to be in a state of plane stress.

Material properties

11. The weight density of the material was assumed to be 150 pcf.* The modulus of elasticity was 4,000,000 psi with a Poisson ratio of 0.20.

Dynamic structural properties

12. Dynamic analysis requires the same input to describe the structural properties as does a static analysis. additional requirements are that the inertia and damping of the structure must be specified.

13. GTSTRUDL will automatically compute member/element inertia contributions by either the lumped or consistent approaches. The member/element weight densities must be provided via the CONSTANTS command or the MATERIALS command prior to a dynamic analysis if automatic computation is to take place. The lumped mass approach is always more computationally efficient and is a reasonable approximation for most problems.

14. Damping is specified in GTSTRUDL in one of two ways depending on whether a modal superposition or direct integration transient analysis is to be performed. In this study, a modal analysis will be performed, thus damping ratios or percent damping would be specified. A 5 percent damping ratio was assumed. Had the stiffness and mass matrices been input via the MATRIX command, damping would have been specified by proportional damping constants.

15. The 5 percent damping ratio is appropriate for a mass concrete dam interacting with a competent rock foundation if the calculated stress levels

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* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

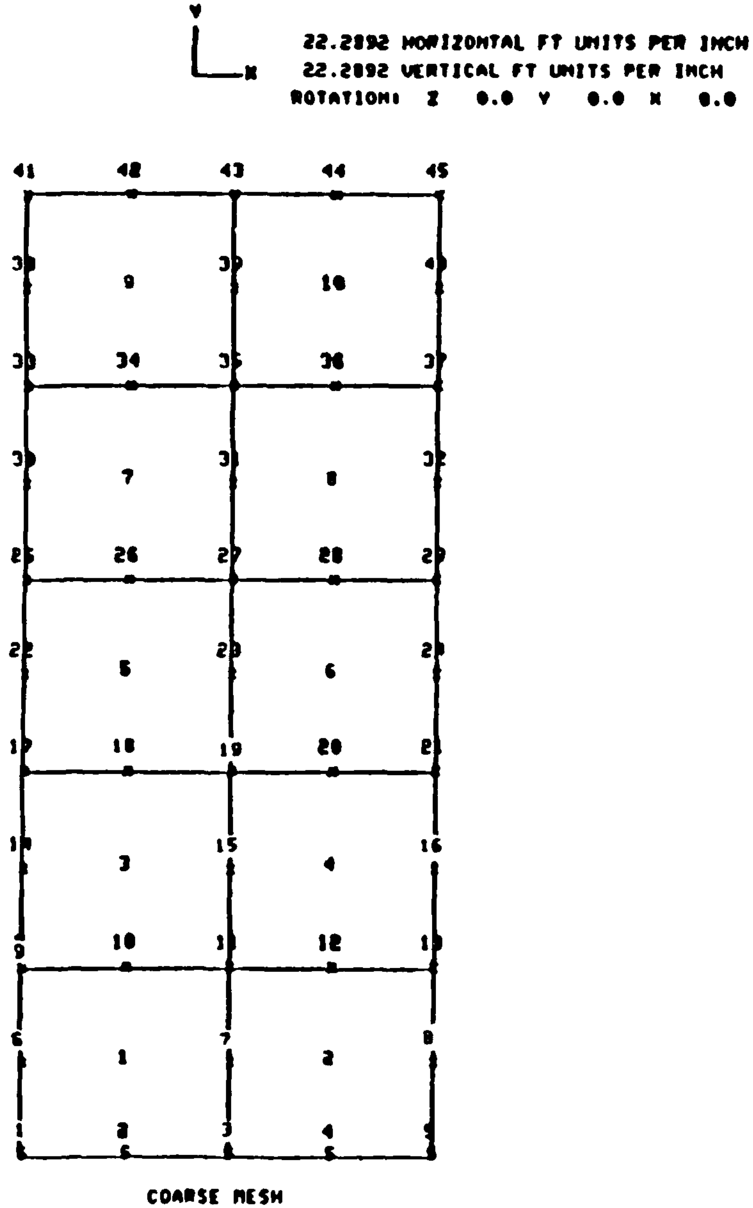


Figure 2. Simplified structure, coarse grid, Mesh 1

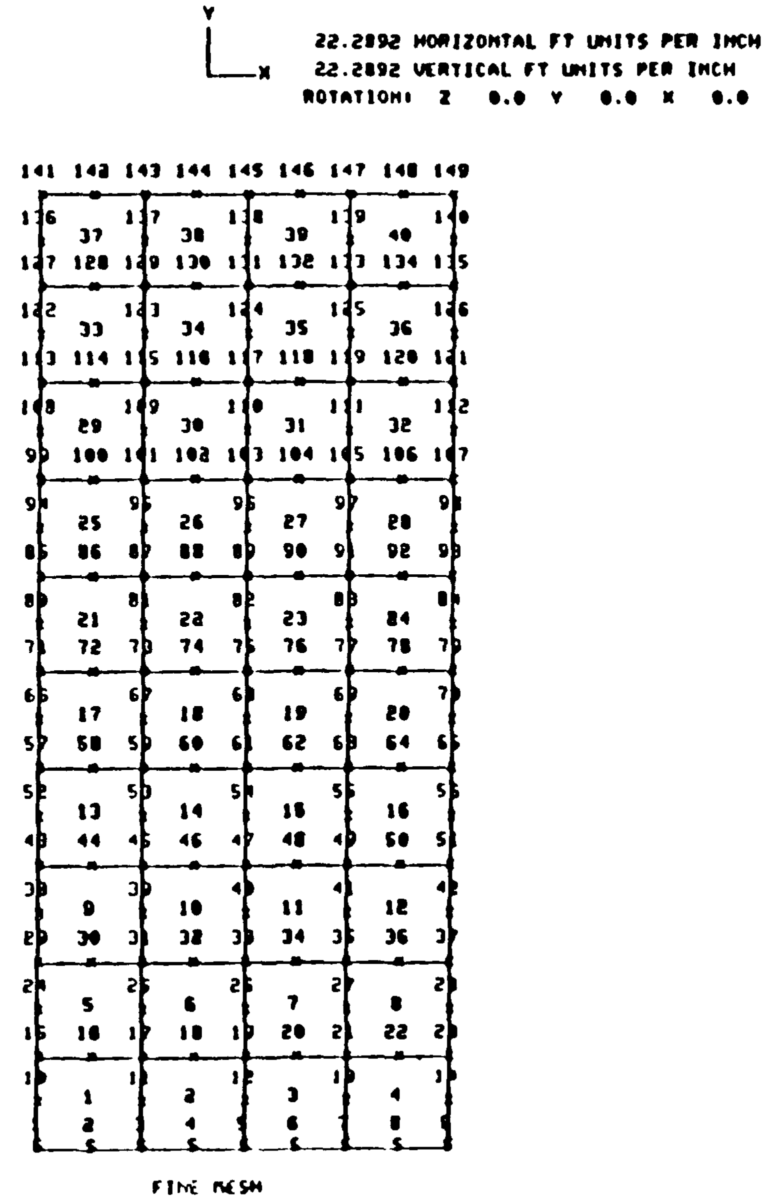
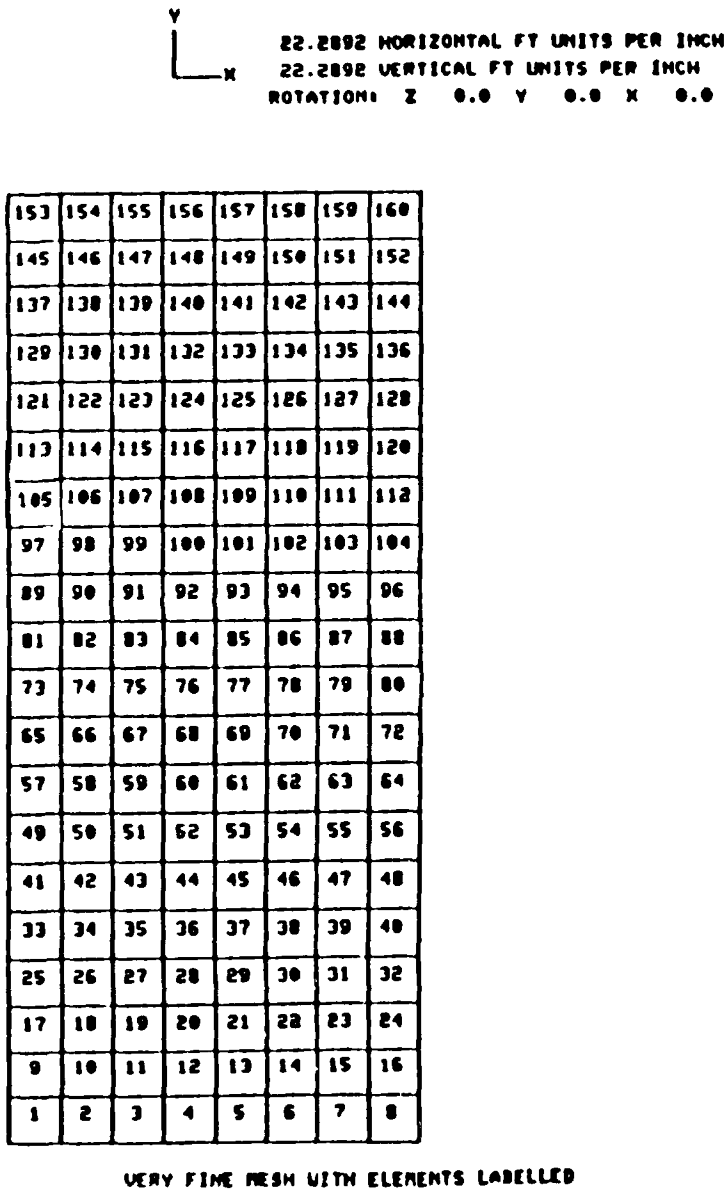


Figure 3. Simplified structure, fine grid, Mesh 2



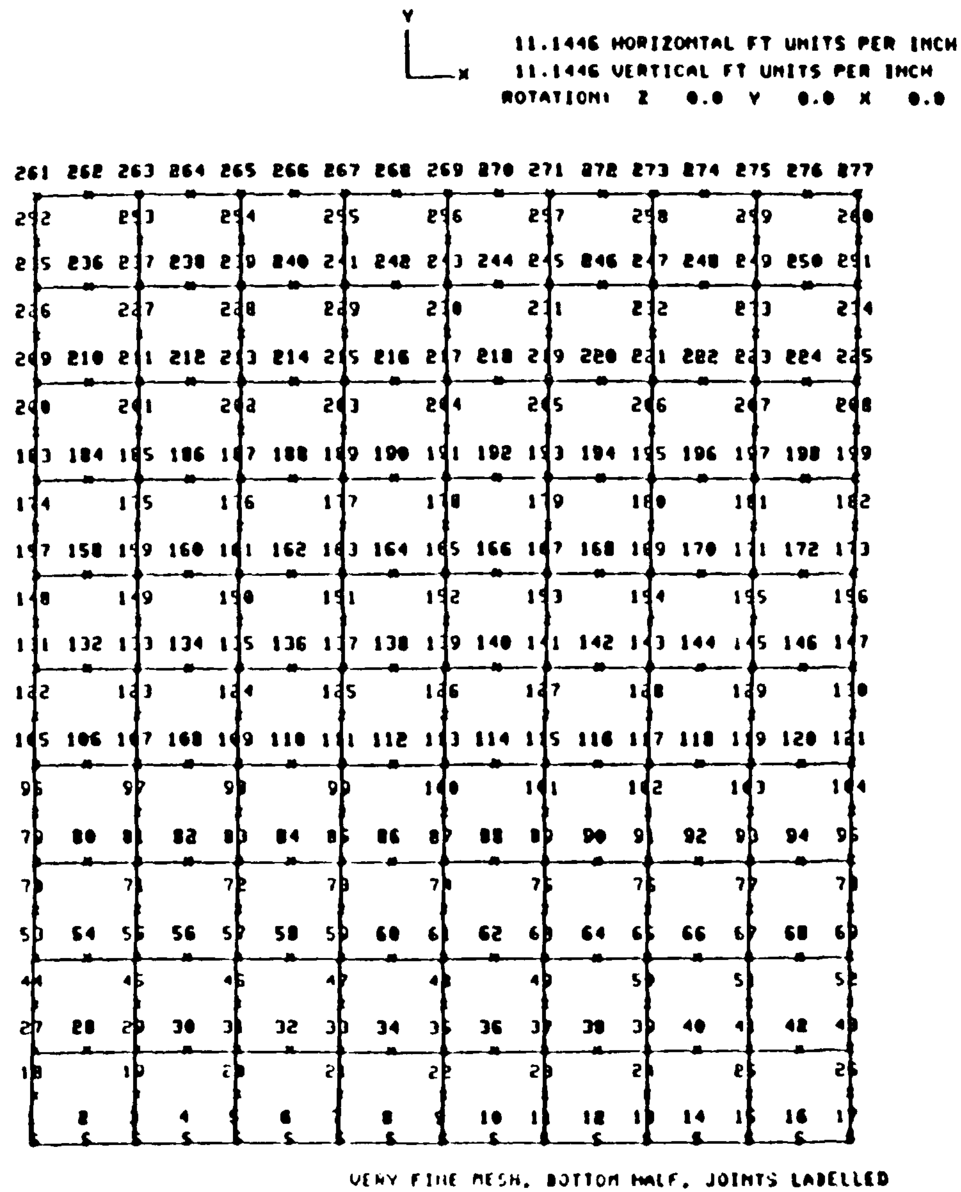


Figure 4. Simplified structure, very fine grid, Mesh 3 (Continued)

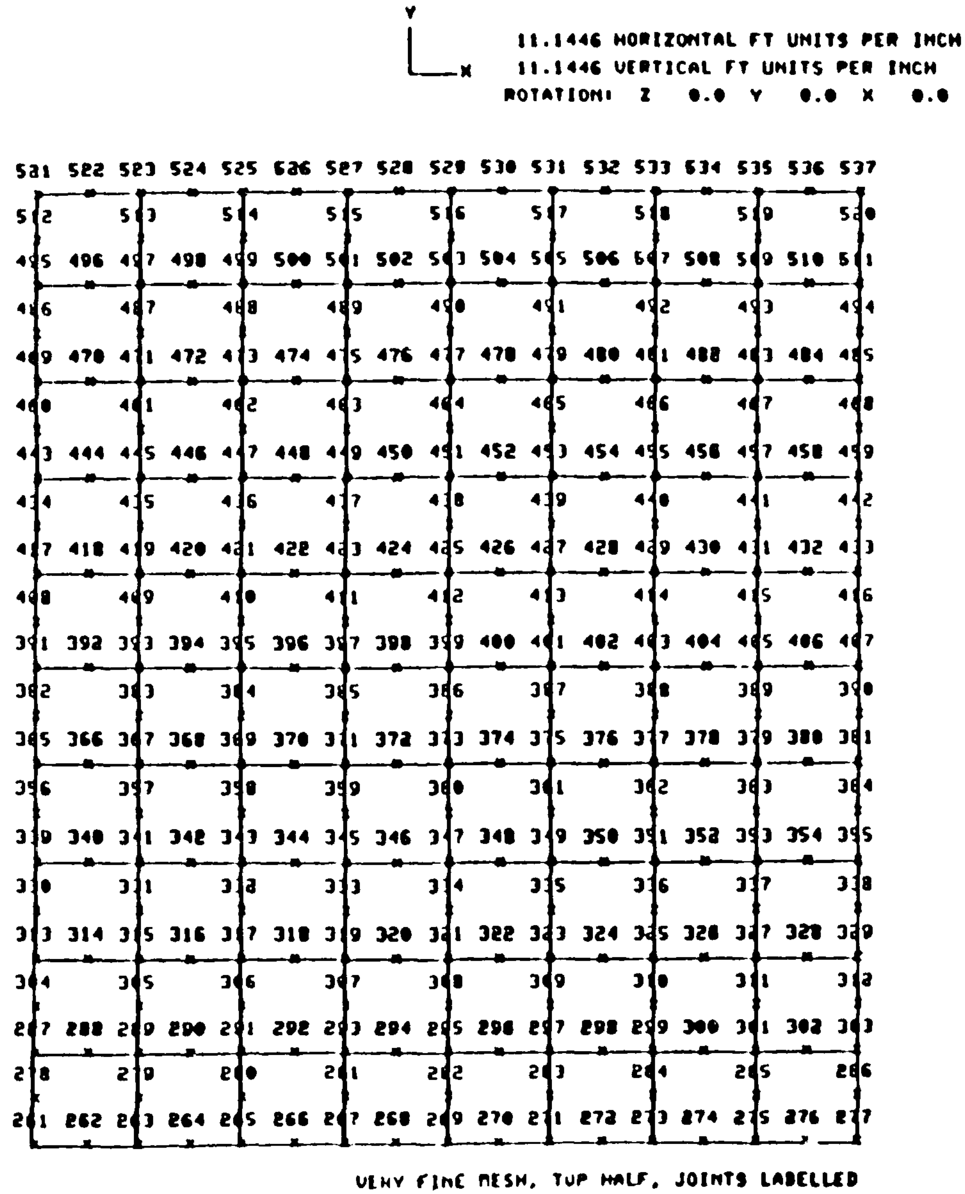


Figure 4. (Concluded)

are within the basic strength capacity of the materials. The damping ratio would be increased if the stress levels go above the basic strength capacity of the material.

Dynamic analysis

16. In performing a dynamic analysis, the GTSTRUDL program first computes the mode shapes and frequencies of the structure. These results are then used to perform a time history analysis or a response spectrum analysis, depending on the choice of the program user.

17. The response spectrum analysis is perhaps the most common technique used in many design offices. The computational effort required for a time-history analysis is often prohibitive, since the response of each selected time point must be computed and stored in order for the maximum response to be identified. Due to this often substantial effort, the response spectrum analysis becomes an attractive alternative technique.

18. Response spectrum analysis is an approximate method of dynamic analysis. It uses the known response of single degree of freedom systems with the same natural frequency and percents of critical damping as the modes of vibration of the structure being analyzed, when it is subjected to the same transient loading.

19. Once the maximum response of each mode is obtained, the maximum total response could be obtained by adding the maximum response of each mode since. In general, however, different modes will attain their maximum values at different times. Therefore, the superposition of the modal maximums will be an upper bound on the actual total response and will significantly overestimate the response for many cases.

20. GTSTRUDL currently computes response spectra maximum responses by combining the modal responses by seven different approaches. ETL 1110-2-303 (Department of the Army 1985) recommends the use of the Complete Quadratic Combination (CQC) method (Der Kiureghian 1980). The CQC method degenerates to the better known Square-Root-of-the-Sum-of-Squares (SRSS) method for simple, 2-D systems in which the frequencies are well separated. Combining modal maxima by the SRSS method can dramatically overestimate or significantly underestimate the dynamic response, especially for 3-D structures.

Design earthquake

21. Professor H. B. Seed's design earthquake response spectrum (Figure 5) (Seed, Ugus, and Lysmer 1974), scaled to 0.25 g peak ground acceleration

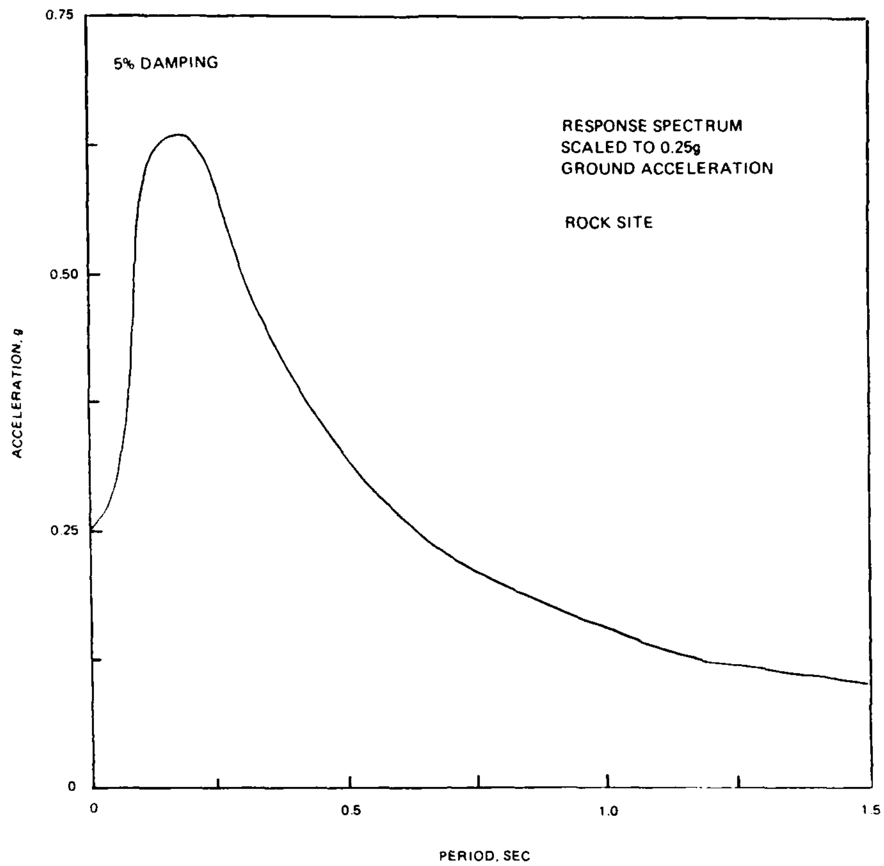


Figure 5. Response spectrum (after Seed, Ugus, and Lysmer 1974)

with 5 percent damping, was used to represent the earthquake for this example.

22. In an actual analysis, the analyst would have to do a geological and seismological investigation of the dam site. The objective of the investigation would be to establish the controlling earthquake, and the corresponding ground motions to be used in the study.

Hydrodynamic effects

23. It has long been understood that the inertial resistance of the water in a reservoir has an important influence on the earthquake response of concrete dams. This study considered a reservoir, or hydrodynamic head of 170 ft on the upstream face of the dam. Except in the case of the EADHI (Chopra and Chakrabarti 1974) and EAGD-84 (Fenves and Chopra 1984) codes, finite element models usually use the concept of an "added mass" or "virtual mass" of water moving with the dam to represent the hydrodynamic interaction effect. There are several methods of approximating this effect. The one chosen for this study is an extension of the Westergaard method that was

originally developed for gravity dams (Department of the Army 1958, Westergaard 1933). Westergaard reasoned that the added mass would produce the same net effect on lateral loads as the parabolic hydrodynamic pressure distribution. The effect is approximated by determining and attaching added masses to the face of the dam. This increased mass results in increased inertial resistance to the motion of the structure when an earthquake is applied and is intended to simulate the actual resistance to the motion of the structure caused by the water mass. Calculation of the added mass may be found in Appendix A.

24. Although research has shown the Westergaard added mass formulation provides a convenient, simple means for representing reservoir interaction in the dynamic analysis of gravity dams, there are limitations that should be noted. The underlying assumptions of Westergaard's work are that the dam is rigid and the water is incompressible. Chopra (1970) has shown that flexibility of the dam and compressibility of water are very important considerations in the dynamic response, hence the development of the EADHI and EAGD-84 codes (Chopra 1970). The main drawback to these programs is that the only dynamic input applicable is an acceleration time-history record. He has therefore developed a simplified response spectrum analysis procedure (Chopra 1978) for use in the analysis of nonoverflow concrete gravity dams and uncontrolled (ungated) spillway monoliths which can be modeled for 2-D analysis. The principles involved are those basic to structural analysis by response spectrum methods. The method represents the hydrodynamic interaction effects by an added mass of water which moves with the dam. However, unlike Westergaard's added mass, this mass is dependent "on the frequency and shape of the fundamental mode of vibration of the dam, and the effects of interaction between the flexible dam and water, considering its compressibility, on the fundamental frequency of the dam."

Input

25. To illustrate the use of GTSTRUDL to solve a problem such as this simplified structure, the input file used for the Mesh 2 model may be found in Appendix B.

Discussion of Results

Mode shapes and frequencies

26. The center-line deflection of the first four mode shapes of the rectangular beam are presented in Figure 6. The actual deflected shapes are shown in Figures 7 to 10, respectively. GTSTRUDL results for the frequencies were compared to Timoshenko beam theory and elementary beam theory results (Table 2). The equations used to generate the results for the closed-form solutions are presented in Appendix C. Timoshenko beam theory included the effects of transverse shear and rotary inertia which is significant due to the relatively short and deep characteristics of the structure.

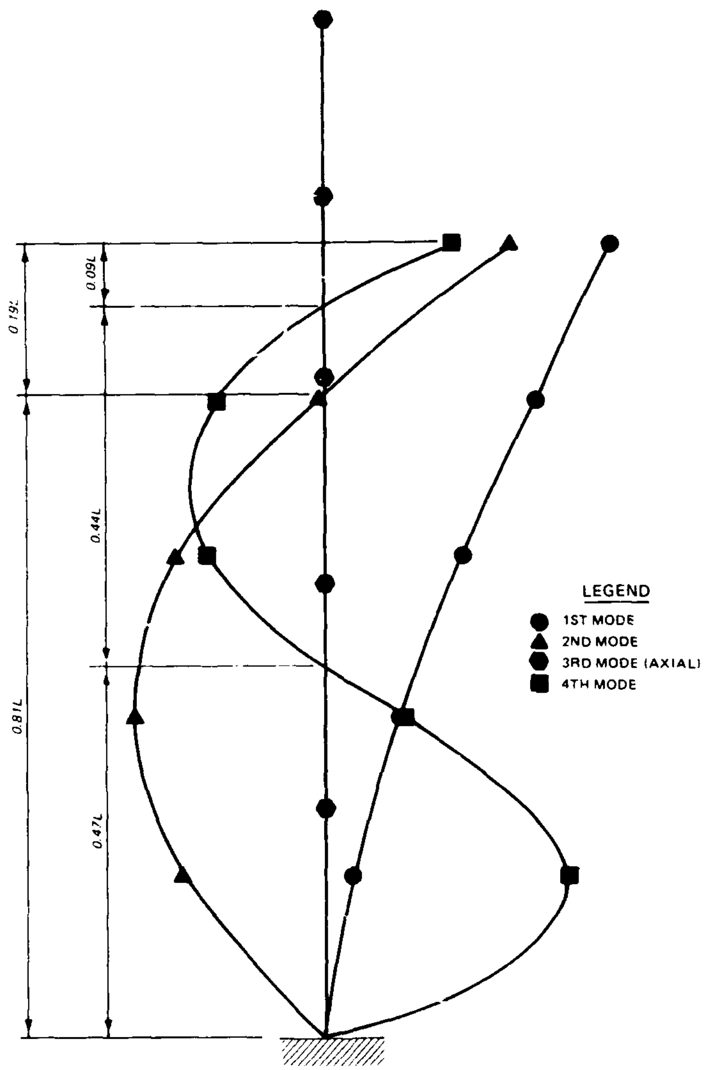


Figure 6. Center-line deflection of mode shapes for simplified structure

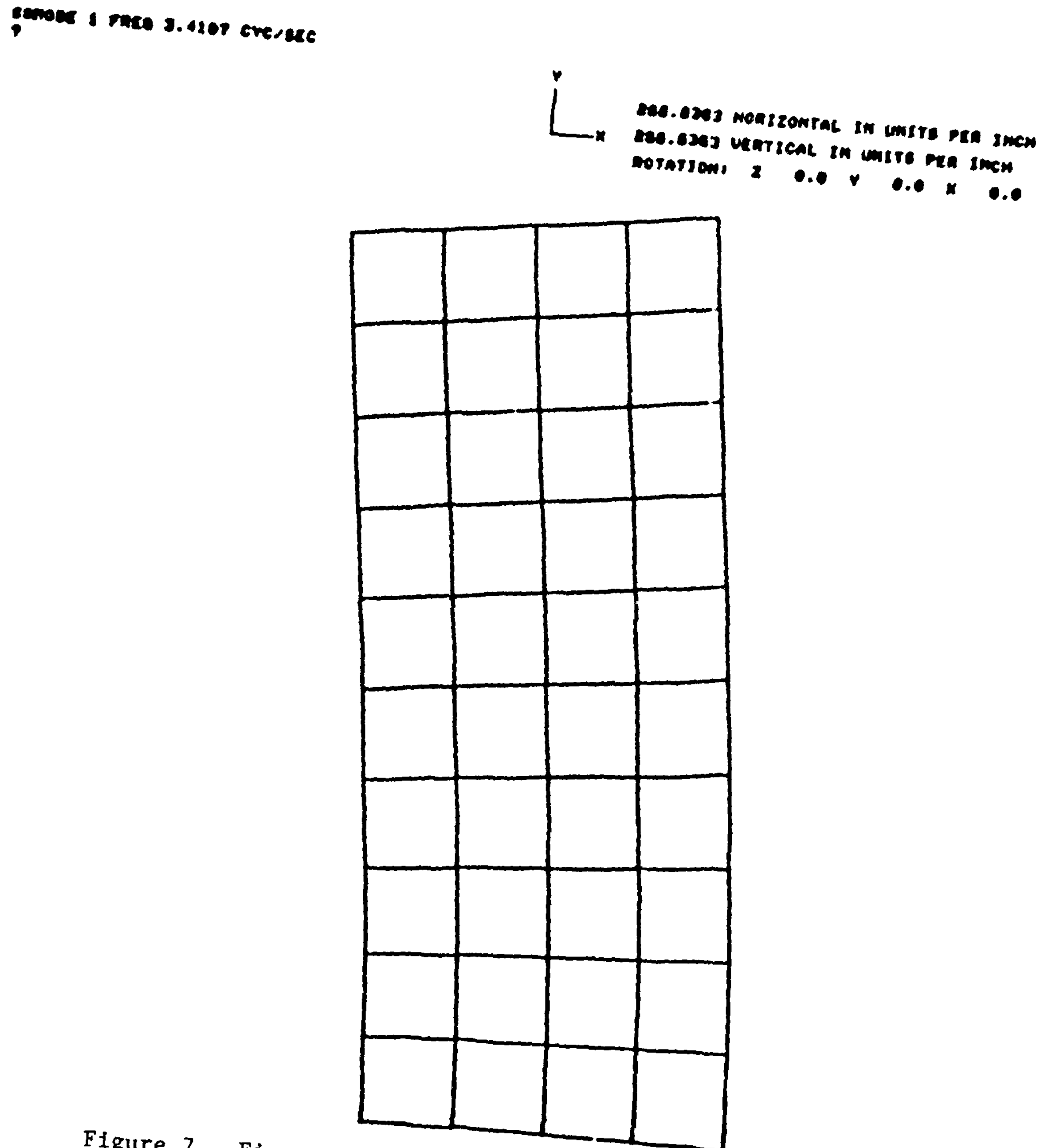


Figure 7. First mode shape of simplified structure

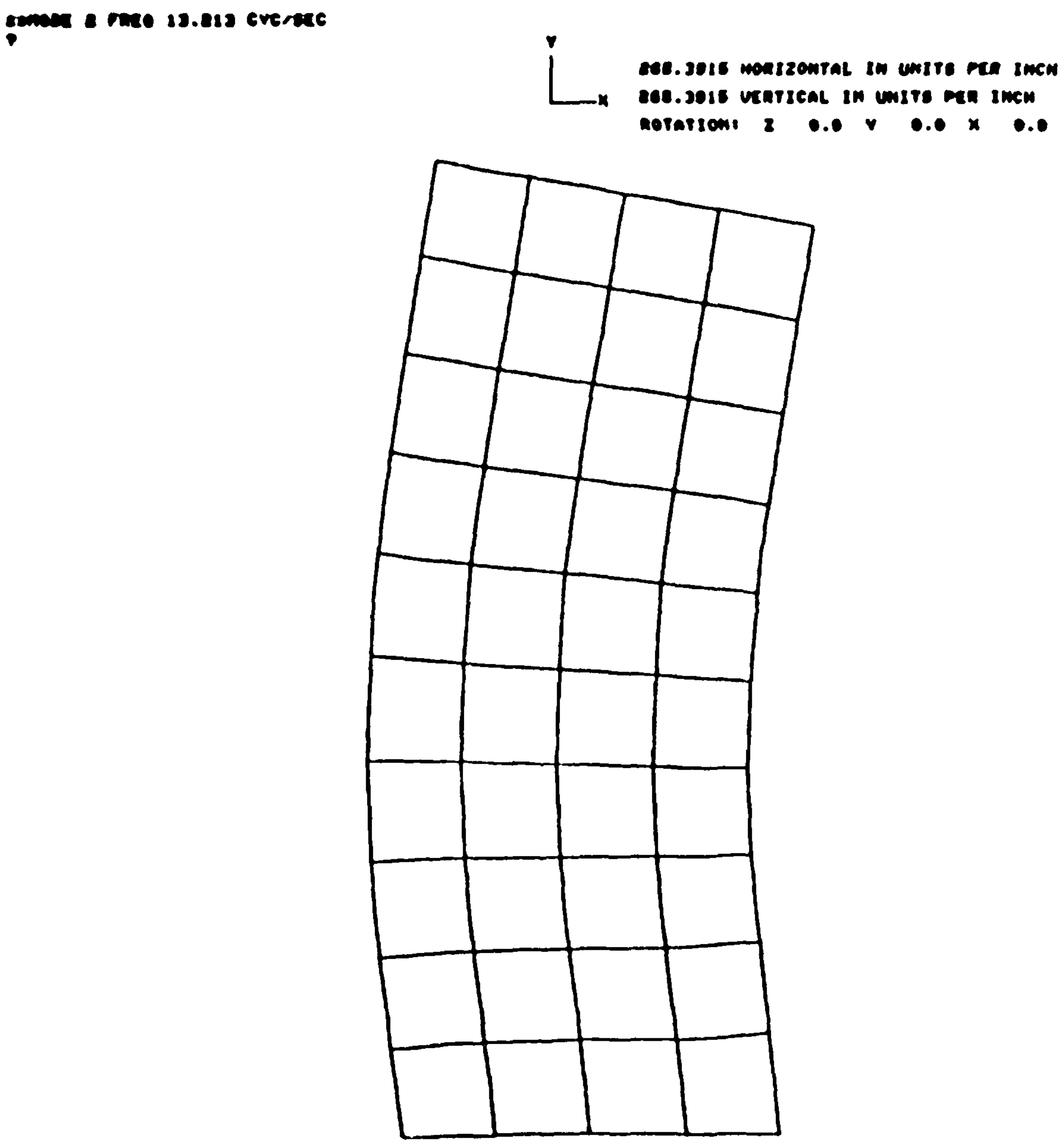


Figure 8. Second mode shape of simplified structure

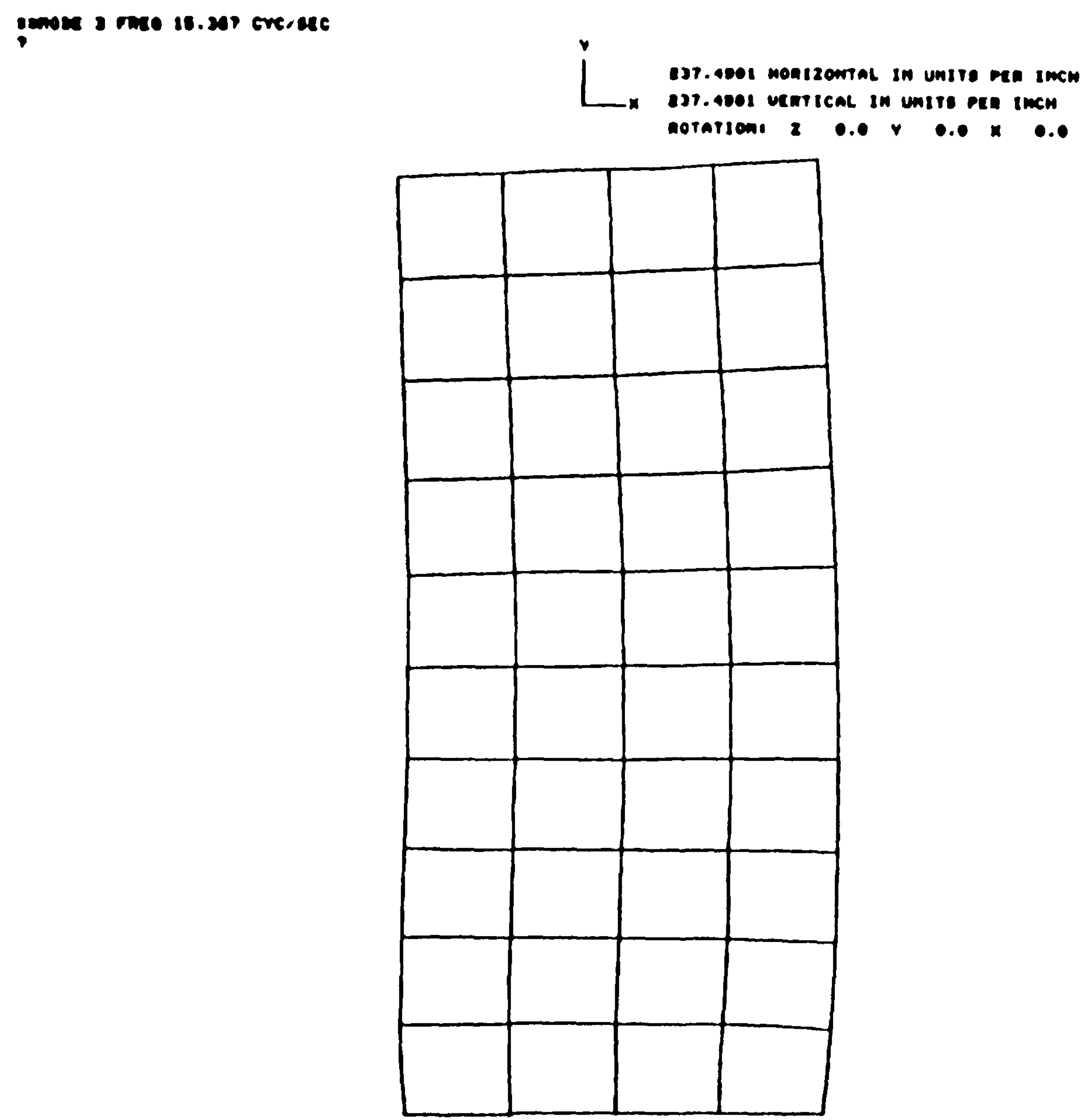


Figure 9. Third mode shape of simplified structure

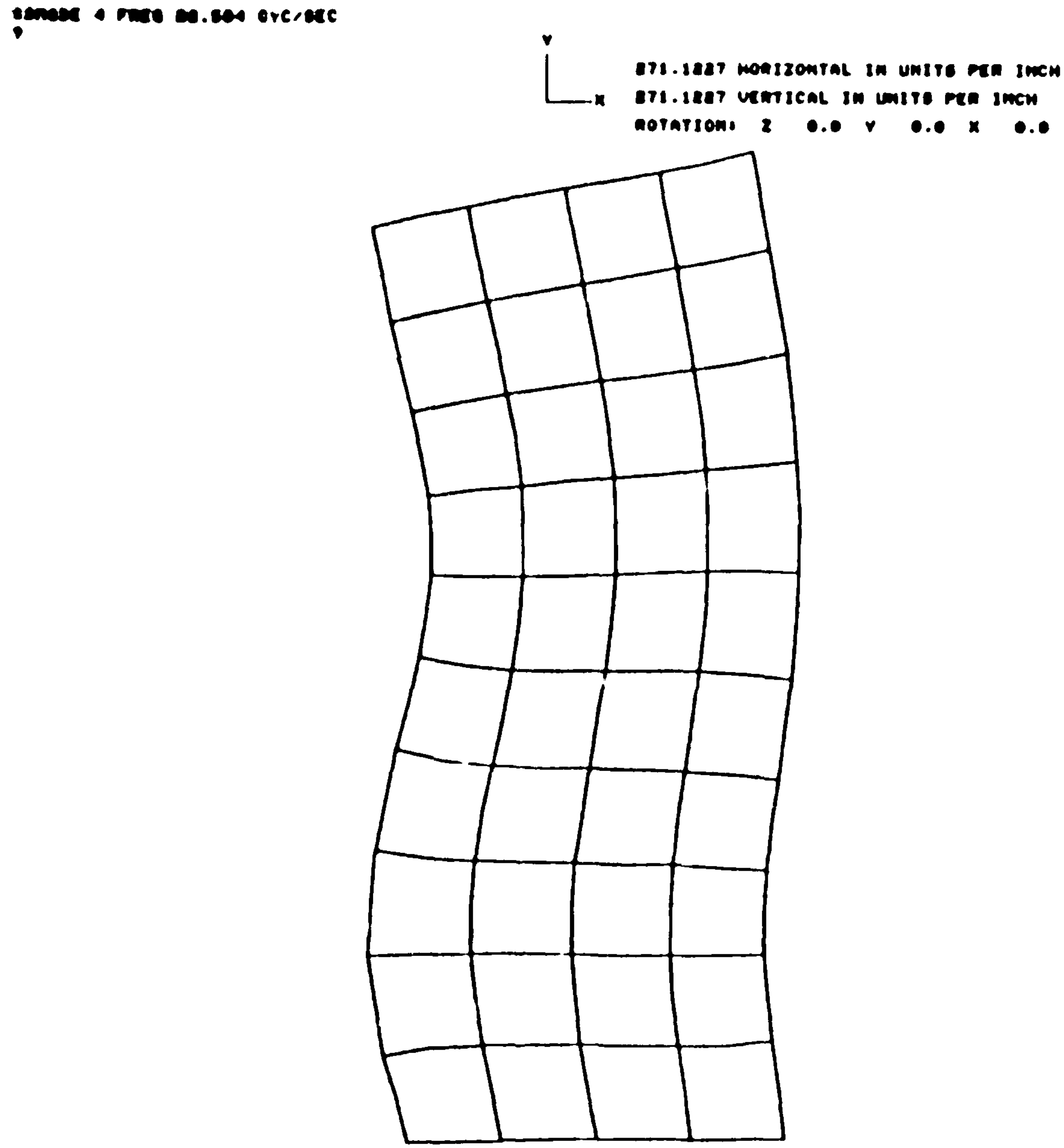


Figure 10. Fourth mode shape of simplified structure

Table 2

Comparison of Frequencies for Simplified Structure

	Frequency, cps						
	GTSTRUDL (Mesh 2)				Includes Added Mass (fixed base) GTSTRUDL†		
Mode	Fixed Base	Pin and Roller*	Elementary Beam**	Timoshenko Beam**	Mesh 1	Mesh 2	Mesh 3
1	3.72	3.63	4.65	4.11	3.34	3.34	3.34
2	15.04	13.52	29.15	16.90	12.90	13.04	13.09
3††	15.37	15.00	--	--	15.03	15.04	15.04
4	32.67	28.19	81.59	35.90	25.82	26.11	26.25

* Although a fixed base should be used in an analysis, a pin and roller base was also run for comparison.

** Appendix C for these results.

† Frequencies of dam including effects of stored water. Other frequencies do not include any stored water effects.

†† This is predominately an axial mode, Figure 6.

27. The second part of Table 2 indicates the deviation in frequencies due to the various model refinements.

Comparison of models for the dynamic analysis

28. A comparison of the deflection results along the height of the rectangular beam is presented in Table 3. The difference in the maximum displacement is approximately 3 percent which indicates a reasonable agreement between the three models.

Table 3

Comparison of Transverse Deflections Along the Simplified Structure

Distance from Base, ft	Deflection, in. Mesh 1	Mesh 2	Mesh 3
18.5	0.021	0.022	0.022
37.0	0.064	0.066	0.066
55.5	0.122	0.125	0.126
74.0	0.193	0.196	0.197
92.5	0.273	0.276	0.277
111.0	0.358	0.362	0.363
129.5	0.446	0.451	0.452
148.0	0.536	0.541	0.543
166.5	0.625	0.630	0.632
185.0	0.712	0.716	0.718

29. A comparison of the SXY (shear) and SYY (normal) stresses was made at two locations along the height of the simplified structure. The results for these stresses for the various meshes at a height of 37.0 ft above the base are presented in Table 4, while those at a height of 111.0 ft are in Table 5.

30. In theory, the finer the mesh the more correct the solution. Therefore, Mesh 3 should be the most correct solution. The results indicate that Meshes 1 and 2 are converging to this solution. For Mesh 1, the difference from Mesh 3 in the maximum stresses is 16 percent for shear and 3 percent for the normal stress. Mesh 2 is within 1 percent in both cases of the results of Mesh 3. Contour plots were obtained for the SXY an SYY stress components for Mesh 2 and are shown in Figures 11 and 12.

31. The analyst, in addition to deciding on the degree of mesh refinement so as to achieve sufficient accuracy, may need to consider cost. The relative costs for the computer runs for Meshes 1, 2, and 3 was $0.81, $3.31, and $23.25, respectively. (These costs are from runs of GTSTRUDL on the Control Data Cooperation Cybernet Computer Service and should be used only as a relative measure.) While Mesh 3 should produce a more accurate solution, the additional cost does not appear to be justified. Mesh 2 provides an acceptable solution, balancing both cost and accuracy.

Model truncation effects

32. An investigation was made to determine the effects of using various numbers of structural vibration modes. Analyses were made using the fundamental mode and subsequently increasing the number of modes until the final solution showed convergence. Results in Table 6 show no difference between modes three and four, thus indicating that only the first three modes are necessary to establish complete convergence. Results also show that there is only a minimal difference in the response using one mode and the computed combined response maximum as represented by the four-mode analysis. This indicates that the first, or fundamental, mode of vibrations participation is predominate in obtaining the response-spectra maximum in this particular dynamic analysis.

Table 4

Comparison of Dynamic Stresses along the Simplified Structure at Height of 37 Ft

	SYY, psi			SXY, psi		
x, ft	Mesh 1	Mesh 2	Mesh 3	Mesh 1	Mesh 2	Mesh 3
-40.0	639	612	618	20	7	5
-30.0		470	468		60	62
-20.0	304	313	313	97	111	107
-10.0		157	156		127	129
0.0	8	4	4	178	141	137
10.0		157	157		127	129
20.0	305	314	314	96	111	105
30.0		470	468		60	62
40.0	638	611	618	20	5	1

Table 5

Comparison of Dynamic Stresses along the Simplified Structure at Height of 111 Ft

	SYY, psi			SXY, psi		
x, ft	Mesh 1	Mesh 2	Mesh 3	Mesh 1	Mesh 2	Mesh 3
-40.0	185	184	184	19	8	7
-30.0		145	145		39	41
-20.0	102	101	101	65	72	69
-10.0		52	52		83	85
0.0	3	3	3	108	93	90
10.0		52	52		82	83
20.0	102	101	101	64	72	70
30.0		147	147		37	39
40.0	188	187	187	18	4	1

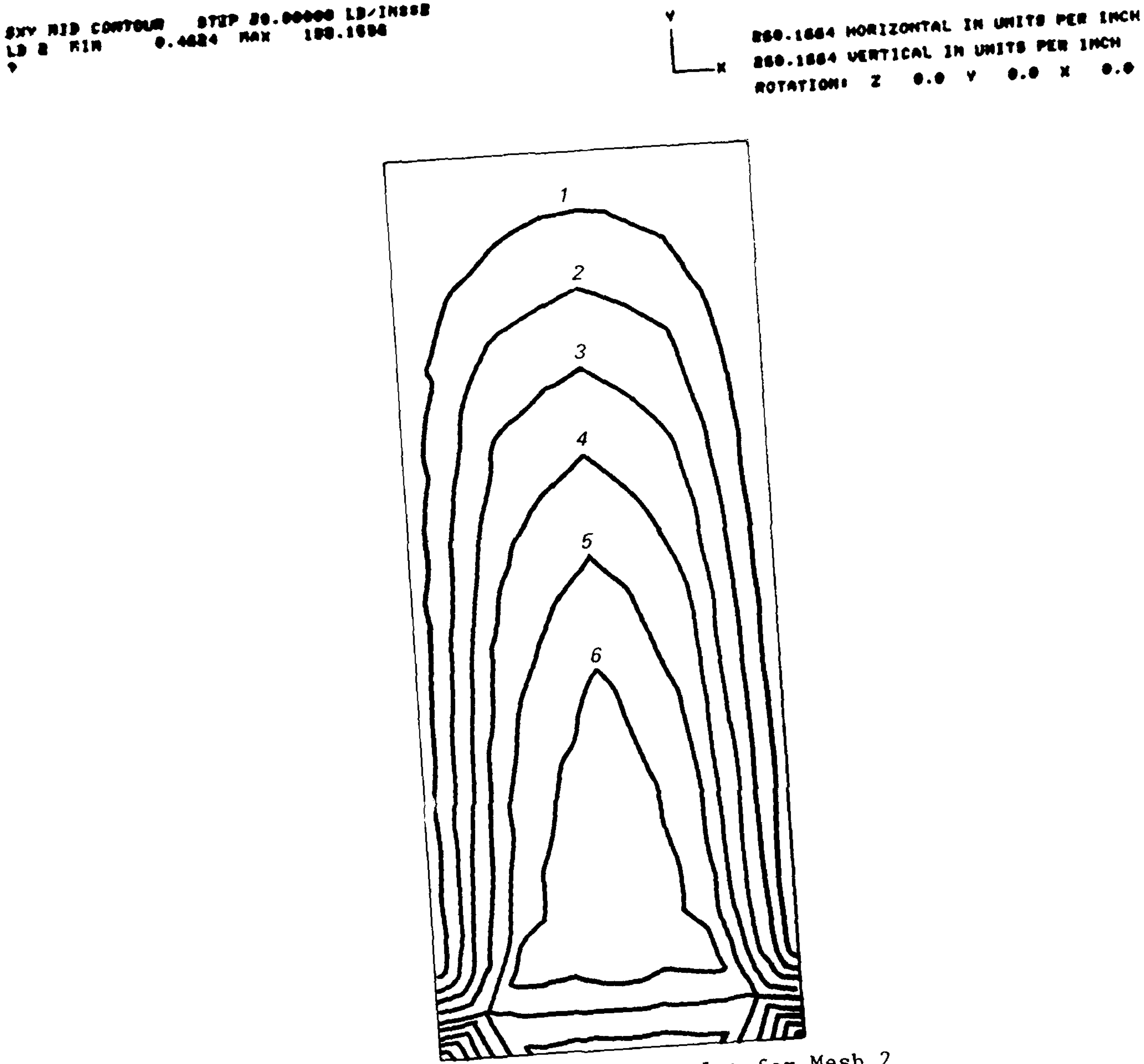


Figure 11. SXY contour plot for Mesh 2

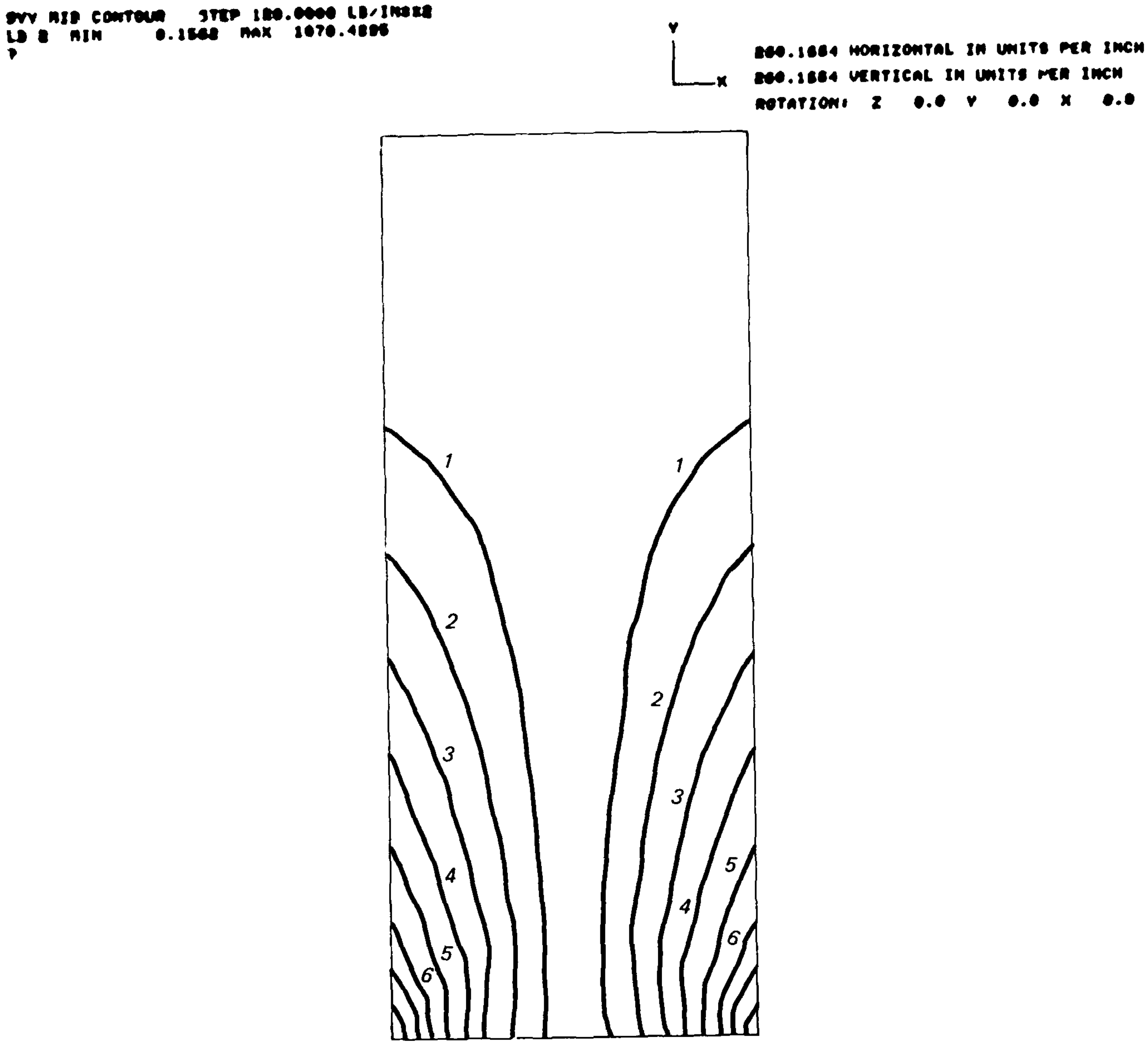


Figure 12. SYY contour plot for Mesh 2

Table 6

Effect of Number of Modes Used in Analysis on Dynamic Stresses (Mesh 2) at Height of 37 Ft

	SYY, psi				SXY, psi			
x, ft	1 Mode	2 Modes	3 Modes	4 Modes	1 Mode	2 Modes	3 Modes	4 Modes
-40.0	611	612	612	612	5	6	7	7
-30.0	470	470	470	470	58	60	60	60
-20.0	313	313	313	313	107	111	111	111
-10.0	157	157	157	157	123	127	127	127
0.0	0	4	4	4	136	141	141	141
10.0	157	157	157	157	122	127	127	127
20.0	313	314	314	314	107	111	111	111
30.0	470	470	470	470	58	60	60	60
40.0	611	611	611	611	5	5	5	5

PART III: GRAVITY DAM EXAMPLE PROBLEM

Description of the Problem

33. An actual earthquake analysis using both the dynamic finite element response spectrum method as outlined in Part II and Chopra's simplified response spectrum method is presented to demonstrate the procedures and illustrate the different results produced by each method.

34. The structure to be analyzed is a nonoverflow monolith similar to those of the Richard B. Russell Dam. The dam is 185 ft high with a reservoir depth of 170 ft. Seed's design response spectrum, scaled to a peak horizontal ground acceleration of 0.25 g with 5 percent damping (Figure 5) (Seed, Ugus, and Lysmer 1974), will be used in both analysis.

35. The geometry of the nonoverflow section is defined in Figure 13.

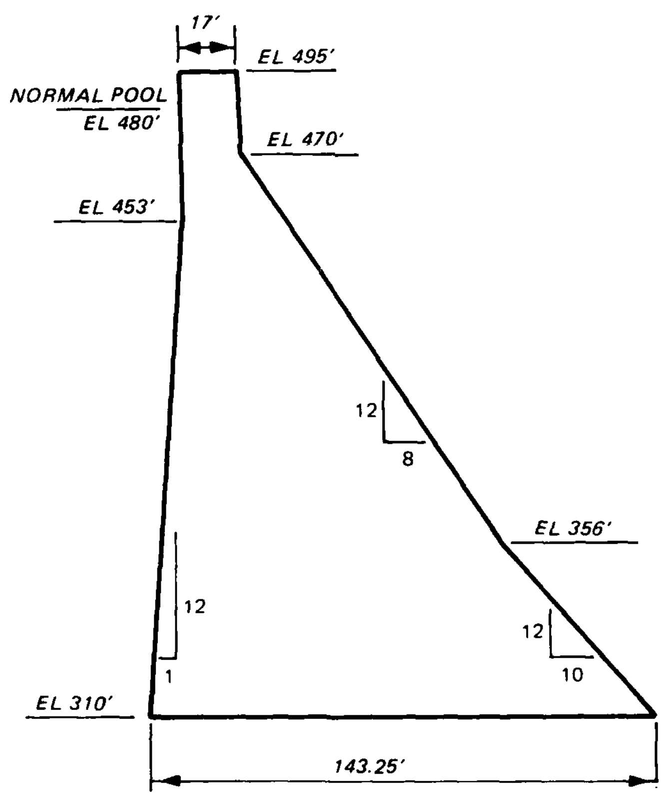


Figure 13. Geometry of dam monolith

Finite Element Method Analysis

36. A listing of the input for the GTSTRUDL finite element analysis of the gravity dam may be found in Appendix D, pages D-2, 3, and 4.

37. Previous results indicated that a mesh with four elements across the base, Figure 14, was a reasonable compromise between accuracy and cost. The mesh contained 36 elements and 135 nodes. The monolith was assumed to be completely restrained along the base.

38. The structure was loaded by hydrostatic and hydrodynamic loadings starting at 170 ft above the base and a self-weight of the concrete of 150 pcf. The hydrostatic pressures were input as uniform edge loads on the upstream elements. The hydrodynamic effect was approximated by attaching Westergaard's (1933) "added masses" to the upstream face nodes, Table 7.

Analysis

39. The analysis is performed in two parts. The static (stiffness) analysis and dynamic analysis are performed separately. These results are then combined to give the final results. The static analysis consisted of two load cases: (1) hydrostatic pressure on the upstream face of the dam, and (2) self-weight (dead load) of concrete.

Results of analysis

40. Results of the independent load cases were obtained. It should be noted that elements incident on a common node will have different stresses at the same node. This is due to the fact that continuity of stresses is not enforced or required for the finite elements in GTSTRUDL, as is true in all other major finite element programs. To obtain a more useful representation of the stresses, one can use the CALCULATE AVERAGE command. To compute the weighted average, GTSTRUDL sums the stresses for all elements incident on a given node, and then divides the sum by the number of elements which are incident on the node.

41. Using the COMBINE command, it is then possible to combine the independent loading conditions to obtain a final result. In this example, one would add the two static loading cases (loads 1 and 2) to obtain a total static loading response (loading combination 5). The static loading condition than can be combined with the dynamic loading.

42. It is first necessary to operate on the dynamic loading (load 3) to transform the results into the form of a static loading condition. The result

GRAVITY DAM
FINITE ELEMENT MODEL

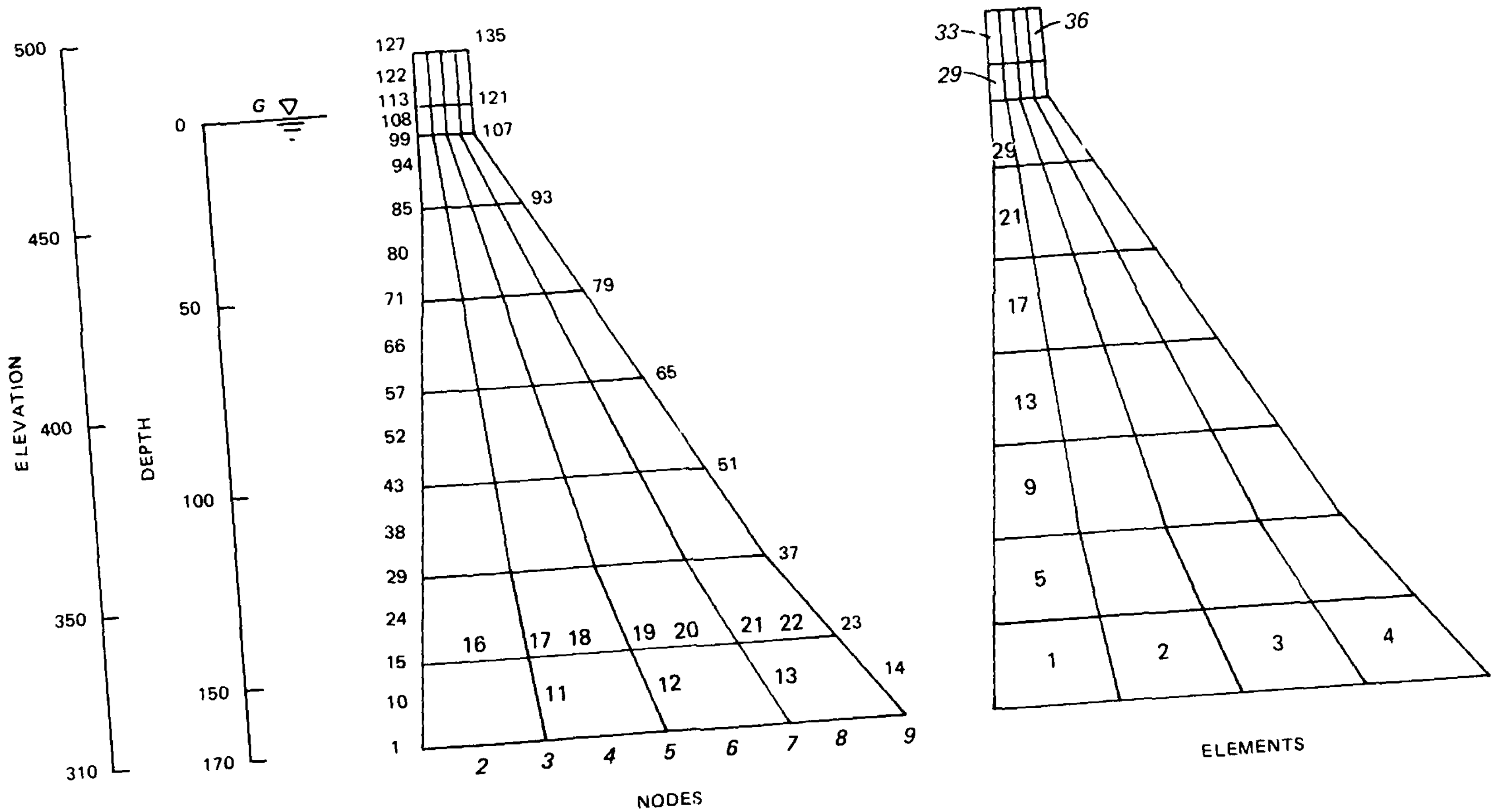


Figure 14. Mesh for dam monolith

Table 7

Structure Loading for Gravity Dam Example

Node	Elevation	Y ft	$\bar{Y}$ ft	y ft	Hydrostatic Pressure psf	Added Mass slugs/ft
127	495.00	185.00				
122						
113	480.00	170.00	0.00		0	55
				2.50		
108			5.00		312	231
				7.50		
99	470.00	160.00	10.00		624	463
				14.25		
94			18.50		1,154	761
				22.75		
85	453.00	143.00	27.00		1,685	1,136
				33.07		
80			39.13		2,442	1,580
				45.19		
71	428.75	118.75	51.25		3,198	1,811
				57.32		
66			63.38		3,955	2,012
				69.44		
57	404.50	94.50	75.50		4,711	2,199
				81.57		
52			87.63		5,468	2,367
				93.69		
43	380.25	70.25	99.75		6,224	2,527
				105.82		
38			111.88		6,981	2,675
				117.94		
29	356.00	46.00	124.00		7,738	2,742
				129.75		
24			135.50		8,455	2,793
				141.25		
15	333.00	23.00	147.00		9,173	2,909
				152.75		
10			158.50		9,890	3,021
				164.25		
1	310.00	0.00	170.00		10,608	1,551

Notes: Y is measured from base of dam (el 310).*
$\bar{Y}$ is depth below water surface (el 480).*
y is depth below water surface to midpoint between nodes.
Ce and Mi are defined in Appendix A.

$$Ce = \frac{51}{\sqrt{1 - 0.72\left(\frac{170}{1{,}000 \times 1.0}\right)^2}} = 51.54$$

$$M_1 = \frac{2 \times 51.54 \times 170^{0.5}}{3 \times 32.2}\left(y_2^{1.5} - y_1^{1.5}\right) = 13.91\left(y_2^{1.5} - y_1^{1.5}\right) = \text{added mass}$$

* All elevations (el) cited herein are in feet referred to the National Geodetic Vertical Datum (NGVD) of 1929.

is called a pseudostatic loading. The command performs this function by copying the results of the specified modal combination (CQC) of the response spectrum loading condition into a static loading condition (loading 4). This can then be added or subtracted with the static loading condition to obtain the maximum or minimum stresses in the dam.

43. A listing of the output can be found in Appendix D. Page D-5 lists the dams first four frequencies. Pages D-6 through D-21 give the stresses for the loading conditions; load 4 - pseudostatic loading, load 5 - static loading, and load 6 - static plus dynamic loading. Extracted from the output and presented in Table 8 are the vertical (SYY) stresses for the monoliths

Table 8

Stresses in Dam

	SYY, psi		
Node	Static	Dynamic	Static plus Dynamic
1	53	313	366
9	-41	44	3
15	-23	275	253
23	-60	88	29
29	-59	260	202
37	-89	167	79
43	-56	261	205
51	-78	192	114
57	-51	260	208
65	-56	197	141
71	-49	257	208
79	-33	199	166
85	-49	273	225
93	-13	194	181
99	-28	188	160
107	-29	285	256
113	-15	79	64
121	-14	54	40

Note: Positive = Tension
Negative = Compression

upstream and downstream face nodes. Column 2 presents the weighted average stress at the various nodes due to the static loading. In Column 3, the dynamic stresses are given. The final column presents the combined stresses, the addition of the static and dynamic stresses. This would represent the maximum tension in the monolith due to the prescribed earthquake with an upstream pool at el 480 ft.

44. The upstream and downstream face node SYY stress component along with the contour plots for the various loading conditions are shown in Figures 15, 16, and 17.

45. As a sidelight to the above example problem, one additional analysis was made. As previously mentioned, the inertial resistance of the water in a reservoir has an influence on the earthquake response of a dam. But, how great is the influence? To answer this, an additional computer run was made without the hydrodynamic effects (attached "added masses"). Figure 18 shows the resulting face node SYY stress components. Comparing this with Figure 16 one can see the higher stresses in the dam due to dam/reservoir interaction.

Chopra Simplified Analysis

46. The more rigorous dynamic analysis of gravity dams is obtained by using finite element computer programs. Due to the capability of these programs to model the horizontal and vertical structural deformations of the dam, to model the exterior and interior concrete, and to include the response of the higher modes of vibration, the interaction effect of the foundation and any surrounding soil, and the horizontal and vertical components of the ground motion, some amount of specialized training is required to use them effectively. Chopra's simplified analysis procedure is a compromise on that complexity.

47. Using a set of standard curves for the fundamental mode shape, the ratio of fundamental period of the dam with and without stored water, and the variation of hydrodynamic pressure over the depth of the water, one can calculate a set of equivalent static lateral loads. These forces are considered to act separately in the upstream and downstream directions, and their effects added to the effects of all other design loads.

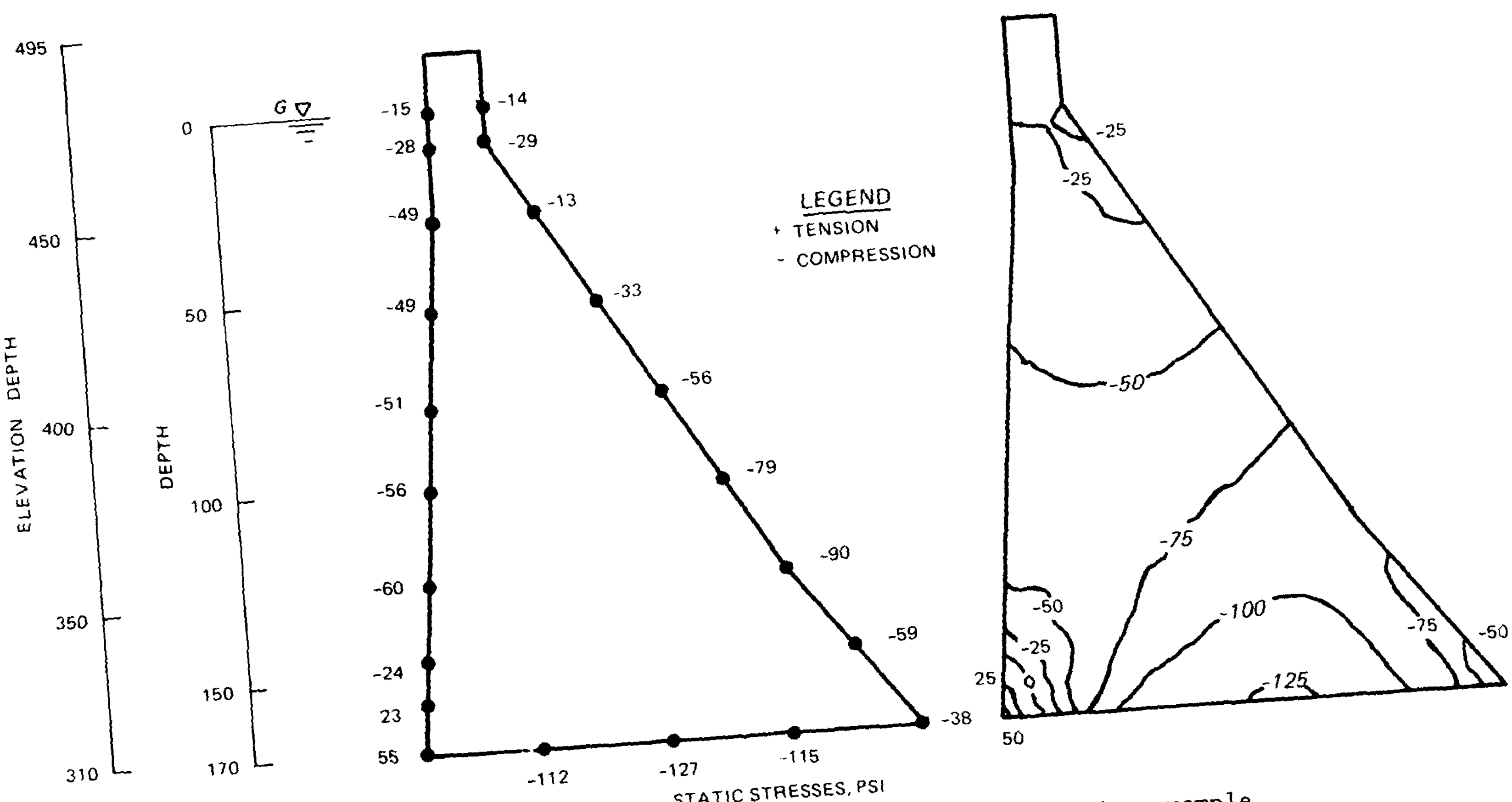


Figure 15. Static stress (SYY) contours for gravity dam example

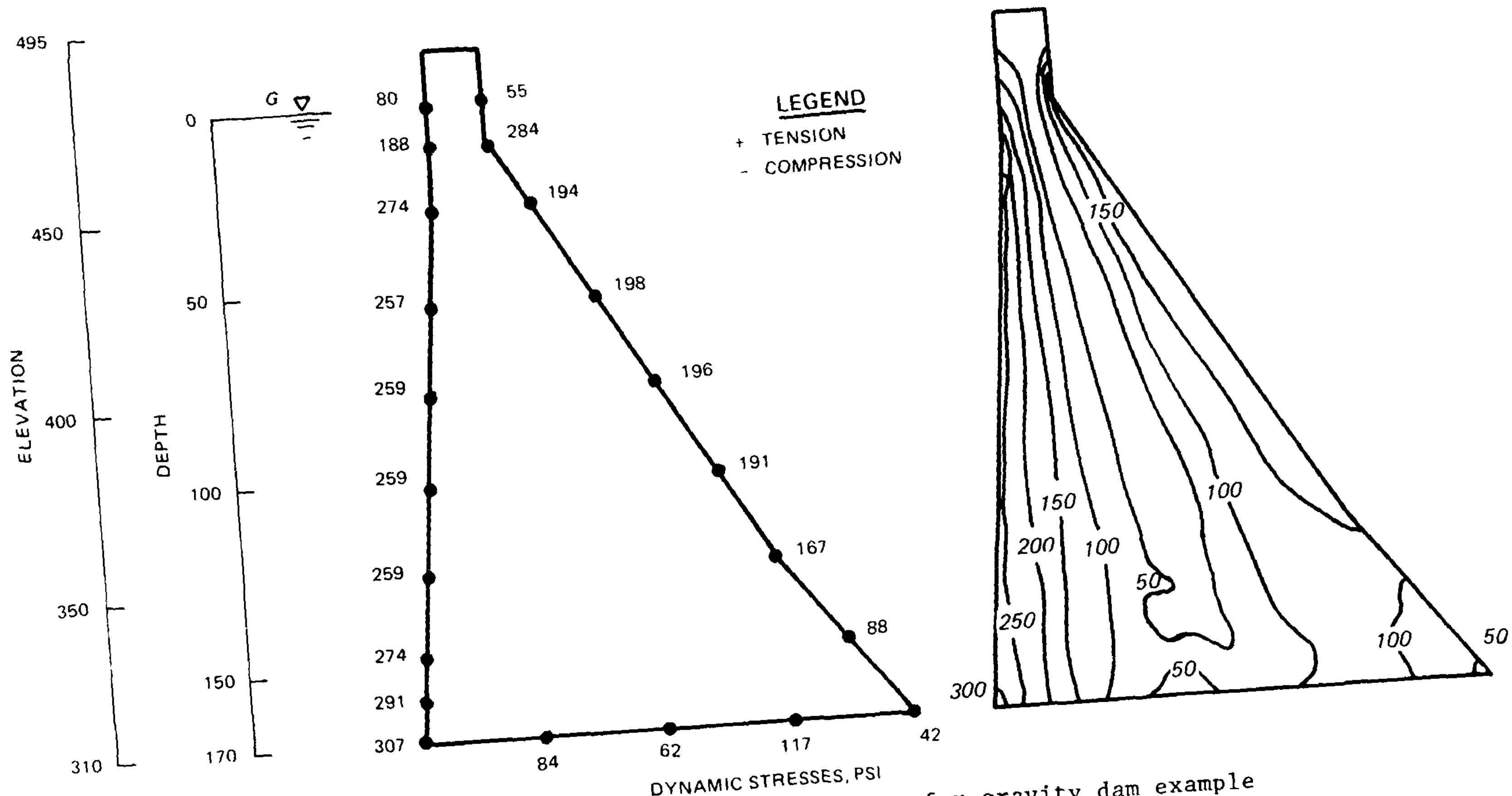


Figure 16. Dynamic stress (SYY) contours for gravity dam example

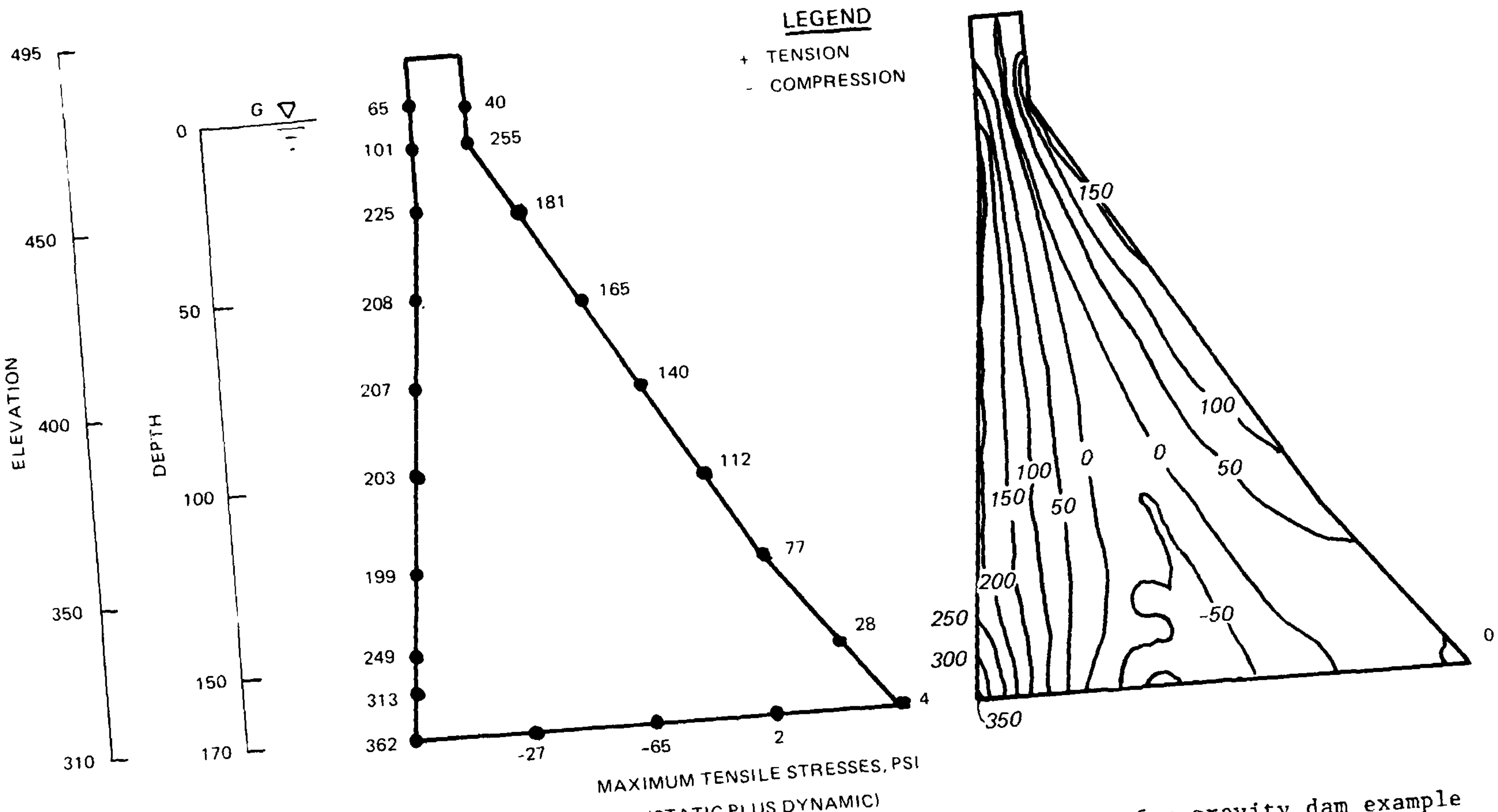


Figure 17. Maximum tensile (static plus dynamic) stress (SYY) contours for gravity dam example

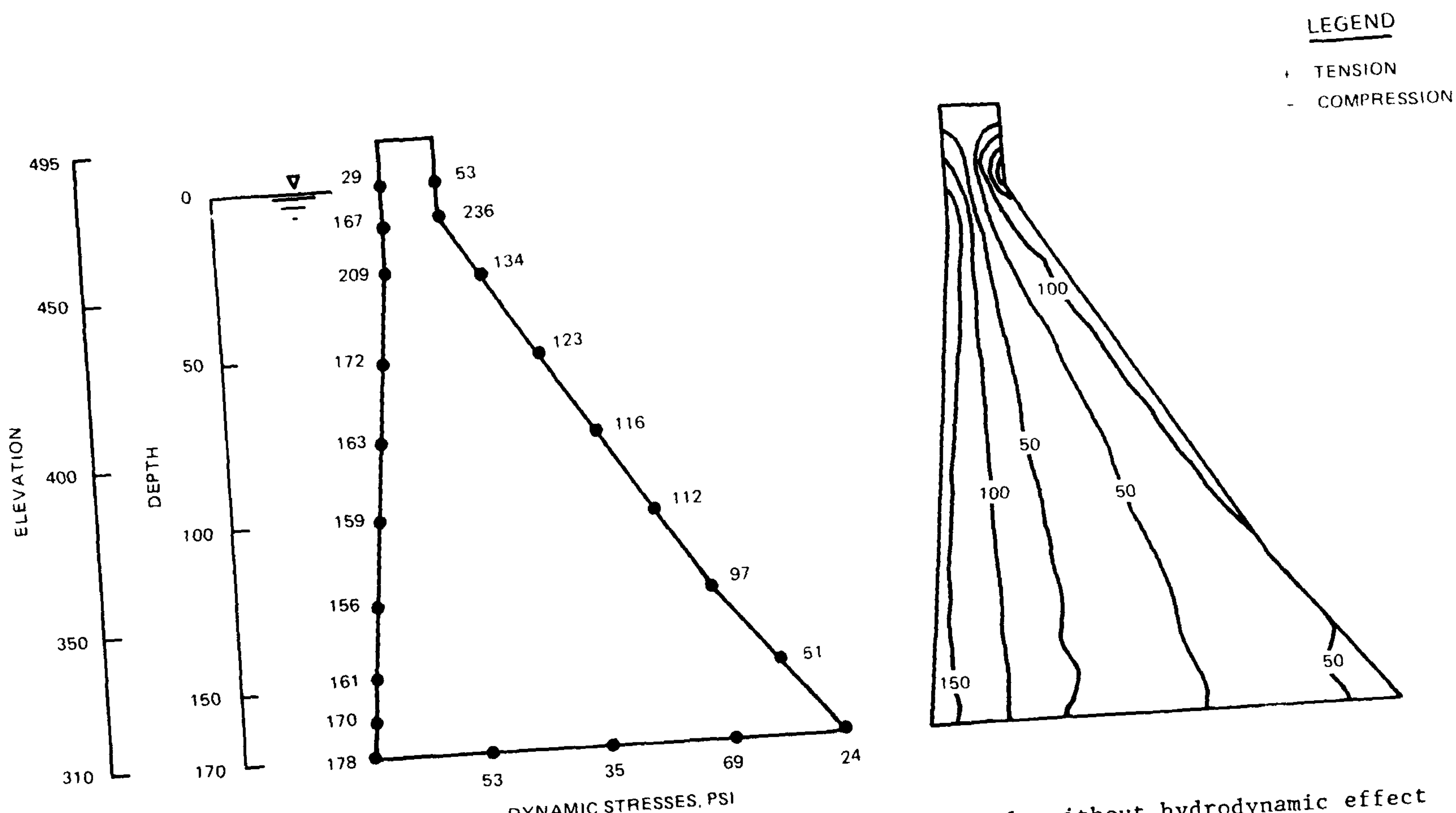


Figure 18. Dynamic stress (SYY) contours for g-avity dam example without hydrodynamic effect

48. The computation of the earthquake forces is carried out as follows:

a. Compute T_s , the fundamental natural period of vibration of the dam, in seconds, without the influence of stored water, from

$$T_s = \frac{1.4 \times H_s}{\sqrt{E}} \tag{1}$$

where

H_s = height of dam, feet

E = modulus of elasticity of concrete, psi

b. Compute $\bar{T}_s$, the fundamental natural period of vibration of the dam, in seconds, with stored water

$$\bar{T}_s = R_1 T_s \tag{2}$$

where R_1 = period ratio determined from Figure 19.

c. Compute R_2 , the ratio of the fundamental resonant period for the impulsive hydrodynamic pressure to $\bar{T}_s$ from

$$R_2 = \frac{4.0 \times H/C}{\bar{T}_s} \tag{3}$$

where

C = velocity of sound in water = 4,720 ft/sec

H = depth of water (feet) in Figure 19

d. Compute $f_s(y)$, the lateral earthquake forces over the height of the dam, including hydrodynamic effects, from

$$f_s(y) = \frac{A_1 \times S_a(\bar{T}_s)}{g}\left[w_s(y)\phi(y) + g\bar{p}_1(y)\right] \tag{4}$$

where

A_1 = scaling constant, with assumed value of 4

$S_a(\bar{T}_s)$ = a spectral acceleration at period of vibration $\bar{T}_s$, from the design response spectrum

g = acceleration due to gravity

$w_s(y)$ = weight per unit height of the dam

$\phi(y)$ = fundamental mode shape factor from Figure 20

$gp_1(y)$ = pressure distribution factor from Figure 21 corresponding to R_2 and multiplied by the quantity $(H/H_s)^2$

e. Compute $f(y)$, the lateral earthquake forces without hydrodynamic effects from:

$$f(y) = A_1 \frac{S_a(T_s)}{g} w_s(y)\phi(y) \qquad (5)$$

where

A_1 = 3

$S_a(T_s)$ = spectral acceleration at period of vibration T_s

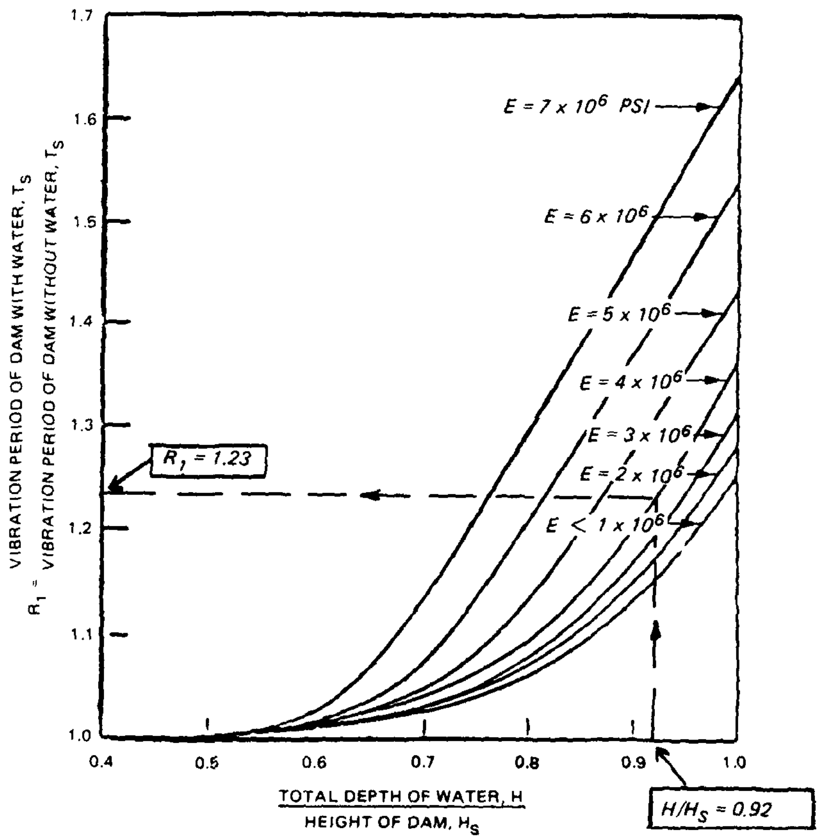


Figure 19. Illustration of how to determine standard values for R_1, the ratio of fundamental vibration periods of the dam with and without water, for a given H/H_s ratio and modulus of elasticity (after Chopra, 1978)

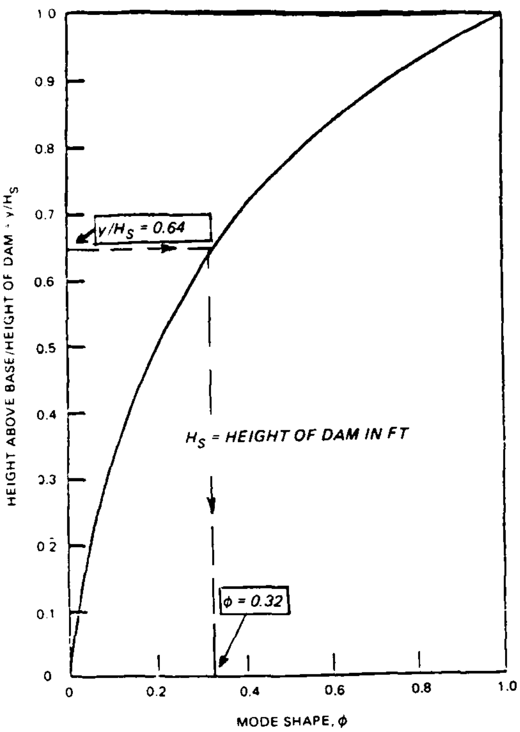


Figure 20. Illustration of how to determine the standard fundamental period and mode shape for a given y/H_s ratio (after Chopra 1978)

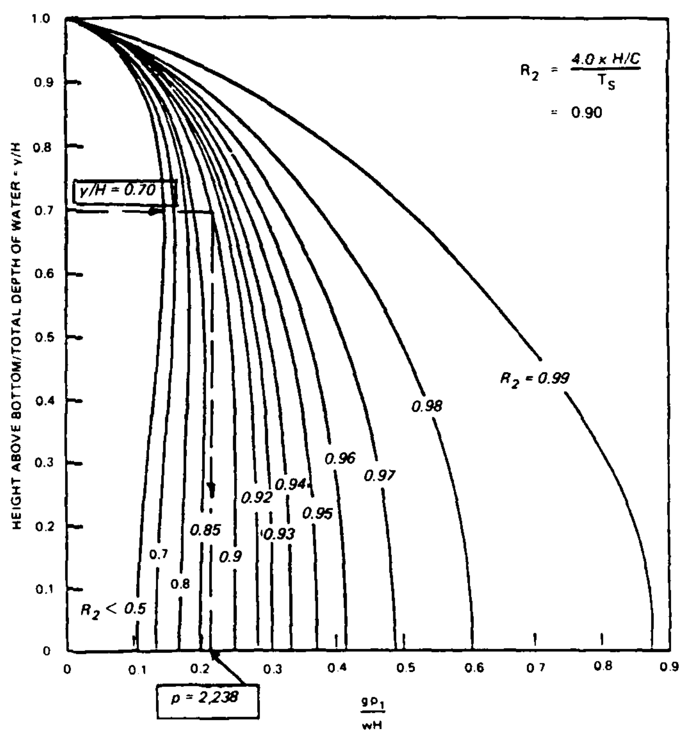


Figure 21. Illustration of how to determine the standard variation of $\overline{gp}_1$ over the depth of water for $H/H_s = 1$ and various values of R_z where W is the unit weight of water (after Chopra 1978)

49. The equations for distributed lateral force (Equations 4 and 5), can be integrated between appropriate limits to yield concentrated forces which are then applied statically to the dam. Stress computations are then carried out as with other design loads.

Computation of earthquake forces

50. For this analysis, the nonoverflow monolith of the gravity dam is divided into nine sections (or horizontal slices). The elevation of each section is equal to the corner node elevations of the elements used in the previous FEM analysis. Thus, section 1 is at an elevation equal to that for nodes 113 through 121 (see Figure 14), section 2 is at nodes 99 through 107, section 3 at nodes 85 through 93, etc.

51. Steps in the computation of the earthquake forces are as follows:

a. For $E = 4.0 \times 10^6$ psi and dam height $H_s = 185$ ft, from Equation 1,

$$T_s = \frac{1.4 \times 185}{4.0 \times 10^6} = 0.13 \text{ sec}$$

b. From Figure 18,

$$R_1 = 1.23 \quad \text{for} \quad E = 4.0 \times 10^6 \text{ psi}$$

and

$$\frac{H}{H_s} = \frac{170}{185} = 0.92$$

From Equation 2,

$$\bar{T}_s = 1.23(0.13) = 0.16 \text{ sec}$$

c. From Equation 3,

$$R_2 = \frac{4.0 \times (170/4,720)}{0.16} = 0.90$$

d. Equation 4 was evaluated along each level throughout the height of the dam, by substituting $A_1 = 4$, and $\left[S_a(\bar{T}_s)/g\right] = 0.63$ (from Figure 5) for $\bar{T}_s = 0.16$ sec, by computing the weight of the dam per unit height, $w_s(y)$, from the section dimensions (Figure 13) and the unit weight of concrete, and by substituting for $\phi(y)$ from Figure 19 and $gp_1(y)$, for Figure 21. The distributed lateral (earthquake) forces $f_s(y)$ and equivalent static load are listed in Table 9 and pictorially shown in Figures 22 and 23.

Computation of stresses

52. At this point, simple beam theory

$$\sigma = \frac{Mc}{I} \pm \frac{P}{A}$$

could be used to calculate the stresses at each horizontal section. As was done with the earthquake forces, the distributed gravity and hydrostatic forces are replaced by concentrated loads. The direct and bending stresses are then computed based on section properties (see Table 10) at each horizontal slice.

53. Tables 11 and 12 show the results of the stress calculations. The earthquake loads have been applied both upstream and downstream and the results combined with the static loads. Figure 24 shows the static stresses (SYY) while Figure 25 shows the maximum tensile stresses (SYY), a result of combining the static and dynamic stresses.

54. An alternative to using simple beam theory to calculate stresses would be to perform a static finite element analysis. This approach has been implemented by Cole and Cheek (Technical Report SL-86-44, Department of the Army) in a new user-friendly computer program. The program was developed using the finite element methods of analysis to determine the dam's inertial response along with Chopra's simplified procedure for estimating the hydrodynamic loading. The program is menu driven, allowing for ease of use by the novice. The only input required is the dam's geometry and appropriate response spectrum. Output consists of principal surface stress values at selected elevations for both the upstream and downstream faces of the dam. A run using this program is included in Appendix E. The results are tabulated in Appendix E, page E4 and shown in Figure 26.

Comparison of Procedures

55. Corresponding calculated stresses are plotted for each method of

Table 9

Equivalent Static Loads to Represent Earthquake Forces for Gravity Dam Example

Section	Elevation	$Y^{(1)}$	$\frac{Y}{H}^{(2)}$	$\frac{g\bar{P}_1}{wH}^{(3)}$	$g\bar{P}_1^{(4)}$	$\left(\frac{H}{H_s}\right)^2 g\bar{P}_1$	$\frac{Y}{H_s}$	$\phi(y)^{(5)}$	$w_s(y)^{(6)}$	$w_s(y)\phi(y)$	$f_s(y)$ (psf)	Equivalent Static Load (lb)
Crest	495.00	185.00	--	--	--	--	1.00	1.00	2,550	2,550	6,426	
												85,785
1	480.00	170.00	1.00	0.00	0	0	0.92	0.78	2,550	1,989	5,012	
												56,675
2	470.00	160.00	0.94	0.098	1,035	874	0.86	0.65	2,550	1,658	6,381	
												129,676
3	453.00	143.00	0.84	0.165	1,755	1,482	0.77	0.48	4,250	2,040	8,375	
												233,588
4	428.75	118.75	0.70	0.211	2,238	1,890	0.64	0.32	6,978	2,233	10,390	
												248,708
5	404.50	94.50	0.56	0.232	2,459	2,076	0.51	0.20	9,705	1,941	10,122	
												238,895
6	380.25	70.25	0.41	0.244	2,588	2,186	0.38	0.13	12,431	1,616	9,581	
												216,456
7	356.00	46.00	0.27	0.248	2,631	2,221	0.25	0.07	15,157	1,061	8,271	
												175,479
8	333.00	23.00	0.14	0.248	2,631	2,221	0.12	0.03	18,367	551	6,988	
												144,463
9	310.00	0.00	0.00	0.247	2,620	2,212	0.00	0.00	21,488	0	5,574	

(1) Y measured from base of dam (el 310).
(2) Depth of water H = 170 ft.
(3) From Figure 20 for $R_2 = 0.90$.
(4) Unit weight of water w = 62.4 pcf.
(5) From Figure 19.
(6) Weight per unit hieght of dam equals section base width (Table 8) times unit weight of concrete (150 pcf).

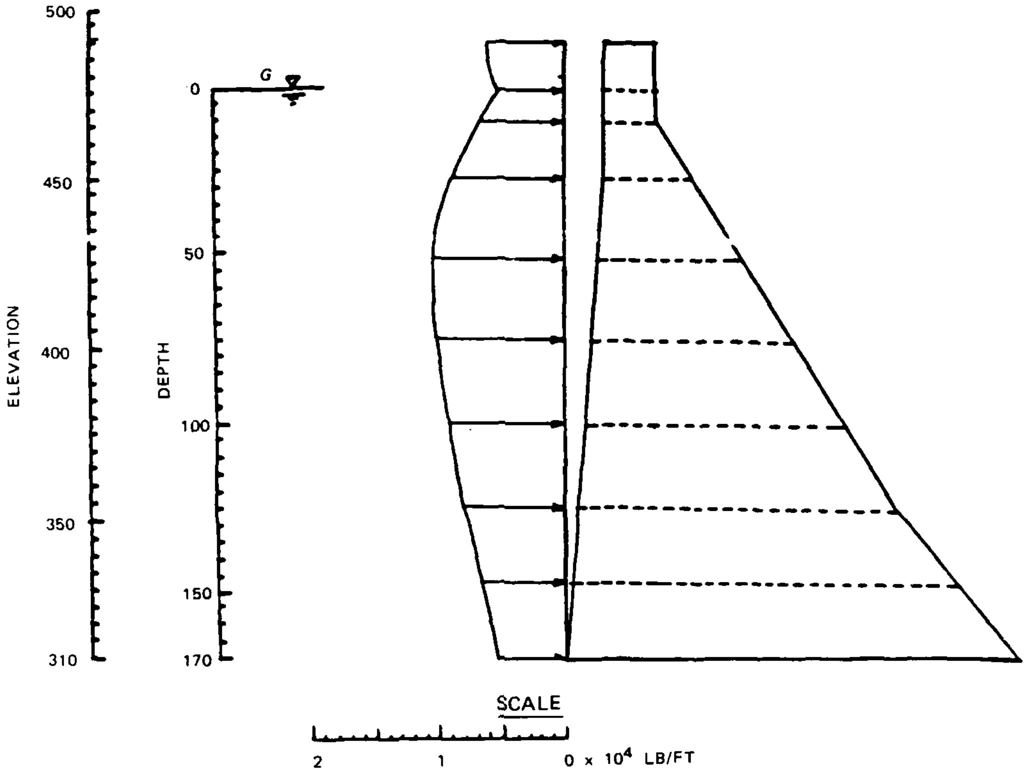


Figure 22. Dynamic force (inertia + water) on gravity dam

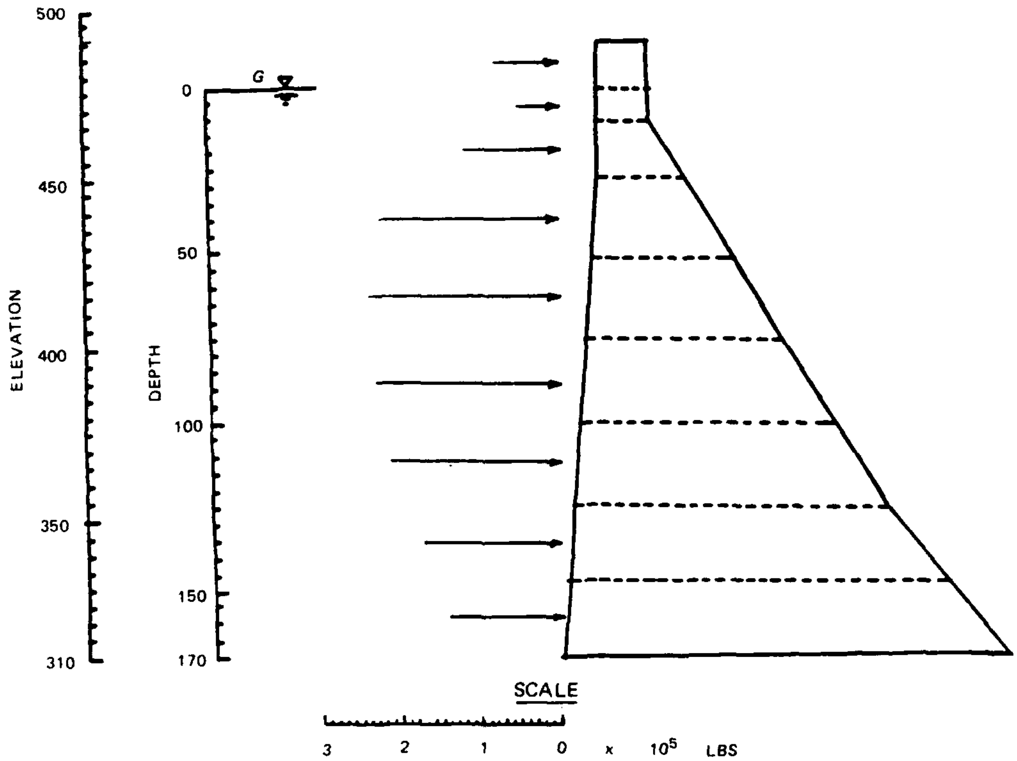


Figure 23. Dynamic load (inertia + water) on gravity dam

Table 10

Section Properties of Gravity Dam

Section	Base Elevation	Base Width ft	W lb	A $in.^2$	S $in.^3$
1	480.00	17.00	38,250	2,448	83,232
2	470.00	17.00	63,750	2,448	83,232
3	453.00	28.33	121,546	4,080	231,254
4	428.75	46.52	257,679	6,699	623,264
5	404.50	64.71	465,435	9,318	1,205,967
6	380.25	82.89	733,883	11,936	1,978,777
7	356.00	101.08	1,068,478	14,556	2,942,544
8	333.00	122.52	1,454,188	17,643	4,323,211
9	310.00	143.25	1,912,641	20,628	5,909,922

analysis; finite element analysis using Westergaard's "added mass" (FEM), Chopra's simplified method using simple beam theory (CSM/SBT), and Chopra's simplified method using finite element analysis (CSM/FEM). Comparisons are presented for both the upstream (Figure 27) and the downstream (Figure 28) faces.

Table 11

Vertical Stresses in Gravity Dam with Earthquake in Downstream Direction Using Simplified Method

Section	Elevation	Static Stresses psi		Dynamic Stresses psi		Static Stresses + Dynamic Stresses psi	
		Upstream	Downstream	Upstream	Downstream	Upstream	Downstream
1	480.00	-16	-16	93	-93	77	-109
2	470.00	-24	-28	257	-257	233	-285
3	453.00	-6	-54	275	-275	269	-329
4	428.75	-30	-46	283	-283	253	-329
5	404.50	-44	-56	299	-299	255	-355
6	380.25	-50	-72	311	-311	261	-383
7	356.00	-52	-94	318	-318	266	-412
8	333.00	-62	-102	299	-299	237	-401
9	310.00	-70	-116	287	-287	217	-403

Note: Positive = Tension.
Negative = Compression.

Table 12

Vertical Stresses in Gravity Dam with Earthquakes in Upstream Direction Using Simplified Method

Section	Elevation	Static Stresses psi		Dynamic Stresses psi		Static Stresses + Dynamic Stresses psi	
		Upstream	Downstream	Upstream	Downstream	Upstream	Downstream
1	480.00	-16	-16	- 93	93	-109	77
2	470.00	-24	-28	-257	257	-281	229
3	453.00	-6	-54	-275	275	-281	221
4	428.75	-30	-46	-283	283	-313	237
5	404.50	-44	-56	-299	299	-343	243
6	380.25	-50	-72	-311	311	-261	239
7	356.00	-52	-94	-318	318	-370	224
8	333.00	-62	-102	-299	299	-361	197
9	310.00	-70	-116	-287	287	-357	171

Note: Positive = Tension.
Negative = Compression.

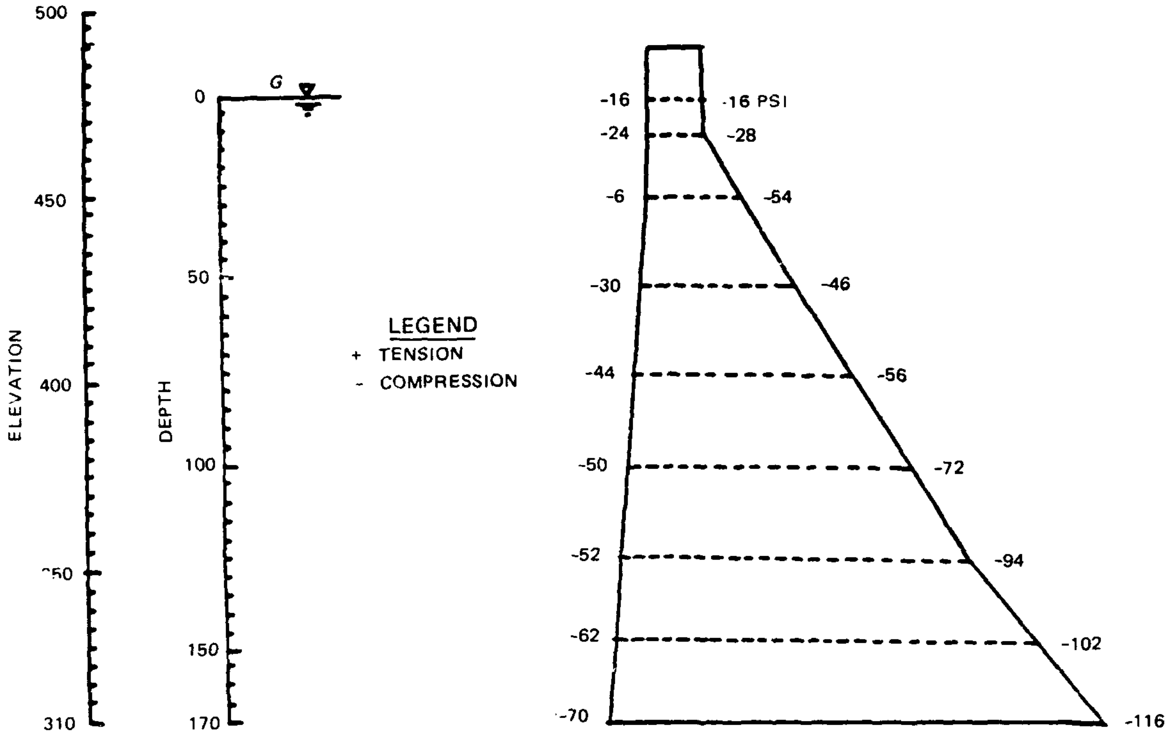


Figure 24. Static stresses (SYY) in gravity dam from simplified method

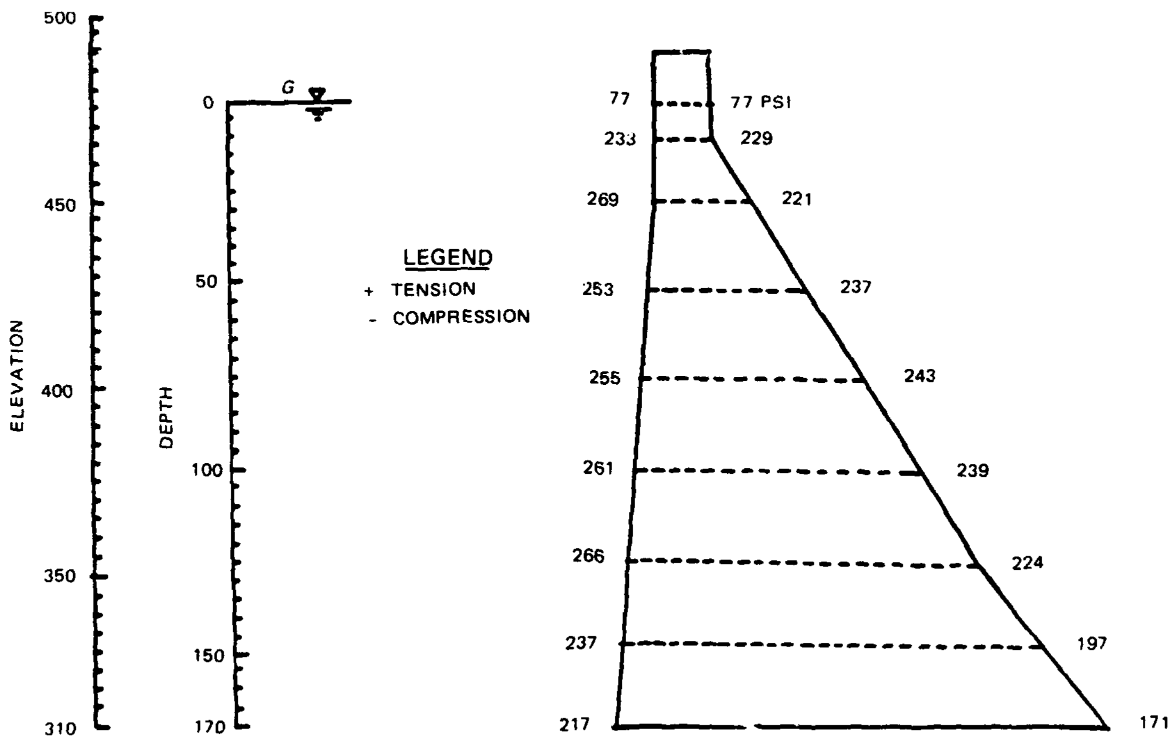


Figure 25. Maximum tensile stresses (SYY) in gravity dam from simplified method

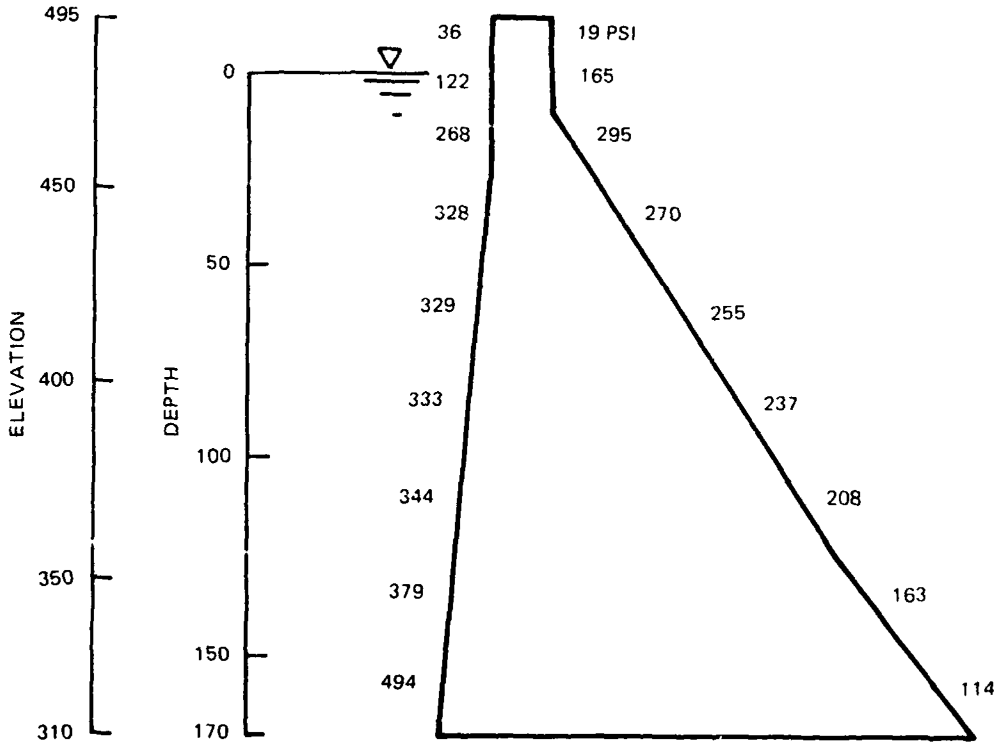


Figure 26. Maximum principal surface stresses

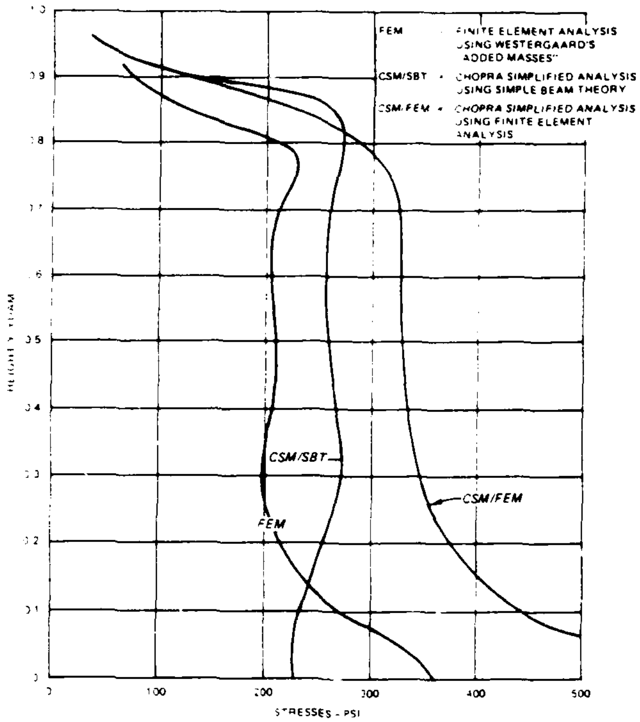


Figure 27. Upstream stresses

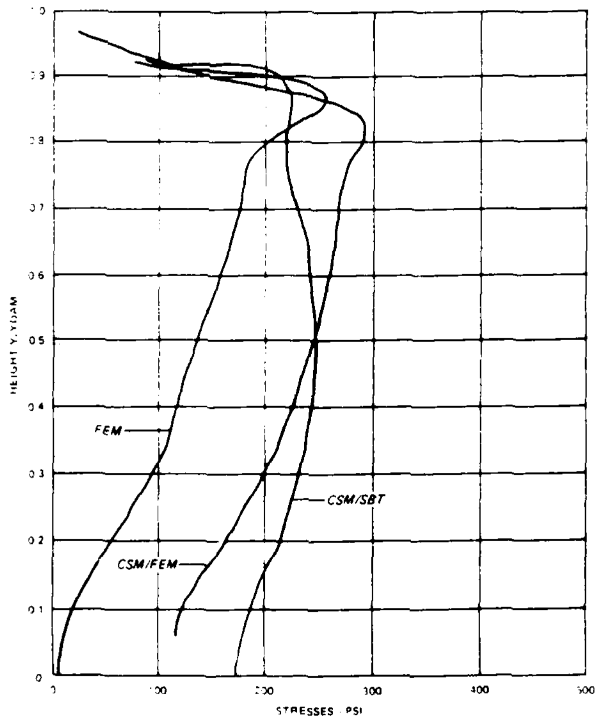


Figure 28. Downstream stresses

PART IV: FOUNDATION EFFECTS

Finite Element Model

56. The purpose of this portion of the study is to determine the necessity of including a foundation in the finite element model used for the dynamic analysis of a gravity dam. The same gravity dam geometry and mesh size as described in the Part III FEM analysis were used for this study with the addition of a foundations block (Figure 29). Limits on the width and depth of a foundation block were evaluated in the Phase Ib report (in preparation). The model used in this study included foundation material in both the upstream and downstream directions equal to the base width of the dam and a depth equal to 1.5 times the base width. The resulting mesh contains 68 elements and 247 nodes.

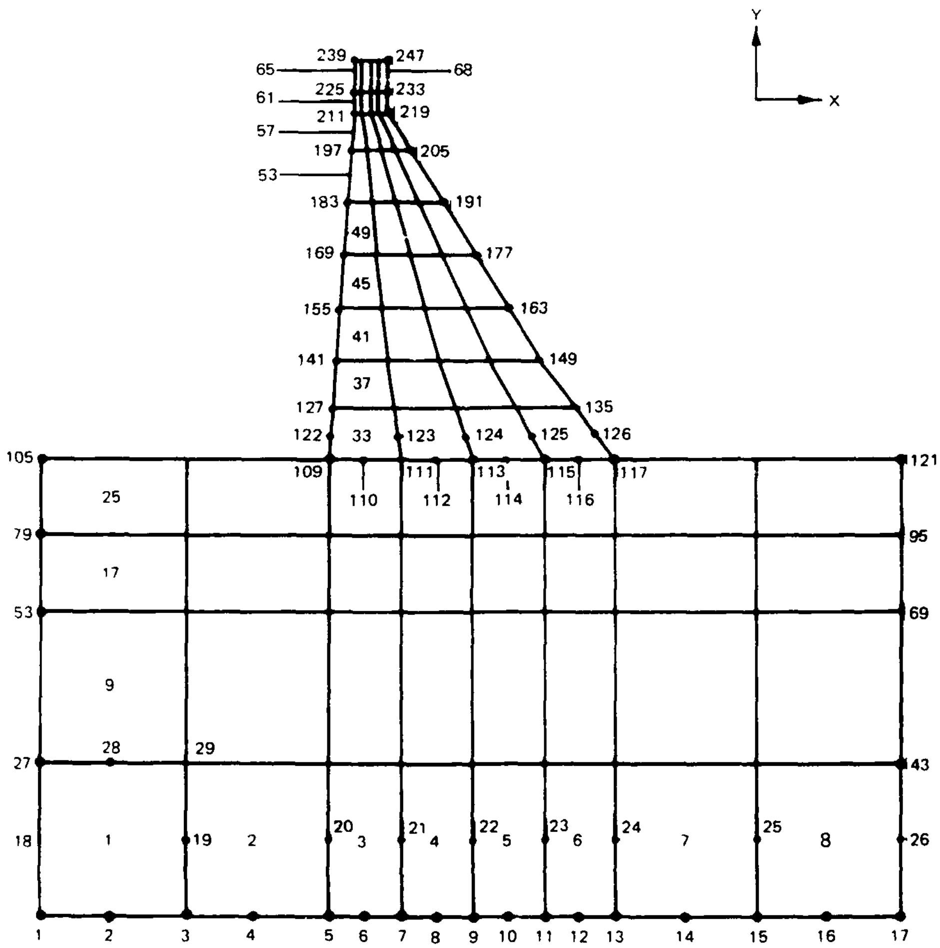


Figure 29. Mesh for dam monolith

57. The foundation boundary was assumed fixed at the corners (nodes 1 and 17) and free to slide along its base and sides. Negligible mass was given to the foundation to avoid wave propagation effects at this level.

Stress Comparison

58. In order to assess the effect of the foundation, the dam was analyzed five times varying only the modulus of elasticity of the foundation material. The various Er/Ec ratios (the ratio of the modules of elasticity of the foundation (Er) to the dam (Ec)) were: 0.05, 0.25, 1.00, 1.75, and 3.00. An analysis assuming an Er/Ec ratio of infinite (∞) is equivalent to the analysis, as performed in Part III, assuming the dam is completely restrained along its base.

59. Comparison of the first mode frequency and period for the various Er/Ec ratios is presented below:

Er/Ec	Frequency (cyc/sec)	Period (sec/cyc)
Infinite	6.042	0.165
3.00	5.543	0.180
1.75	5.256	0.190
1.00	4.836	0.207
0.25	3.353	0.298
0.05	1.731	0.578

The addition of a foundation to a model, regardless of its size, will decrease its frequency and increase its period. This substantiates that in addition, as the foundation's modules of elasticity decrease, it will further reduce the frequency and increase the period.

60. The upstream and downstream dam face node SYY stress components along with the contour plots for the static, dynamic, and combined static and dynamic analysis are obtained for the various Er/Ec ratios and shown in Figures 30 through 44. Previously presented in Figures 15, 16, and 17 were the stress components and contour plots for the analysis assuming a completely restrained base (Er/Ec = infinite). Combined results for the SYY stress components are presented in Figures 45 through 50.

61. Figures 45 through 46 show that as the foundation's modulus of

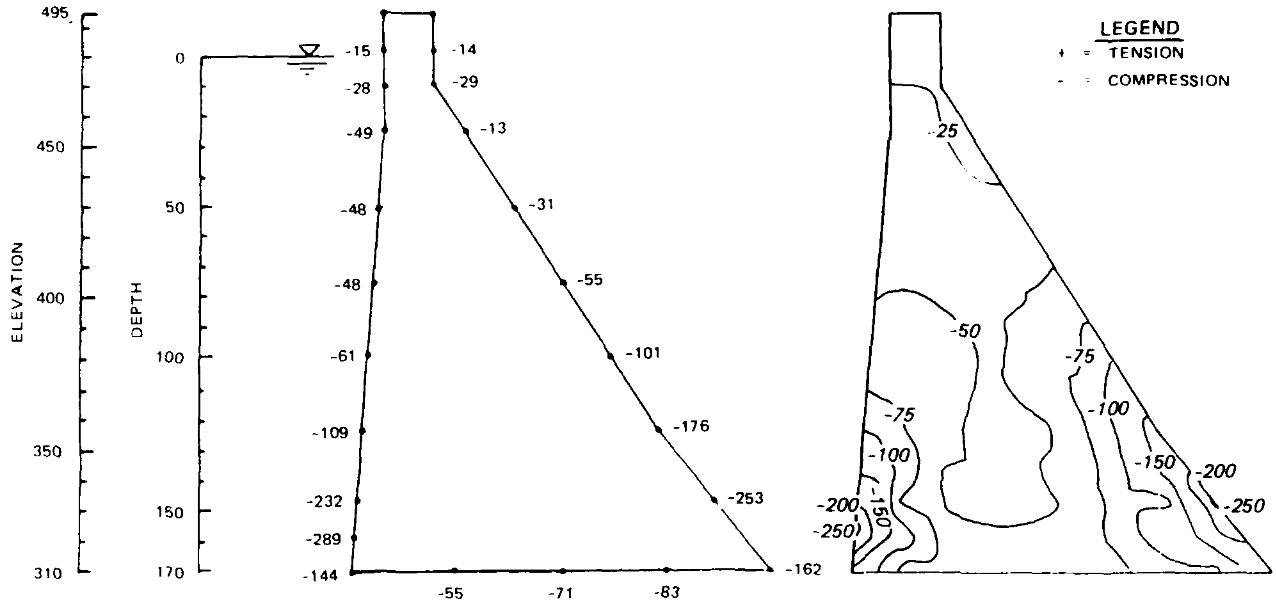


Figure 30. Static stresses (SYY) for Er/Ec ratio of 0.05

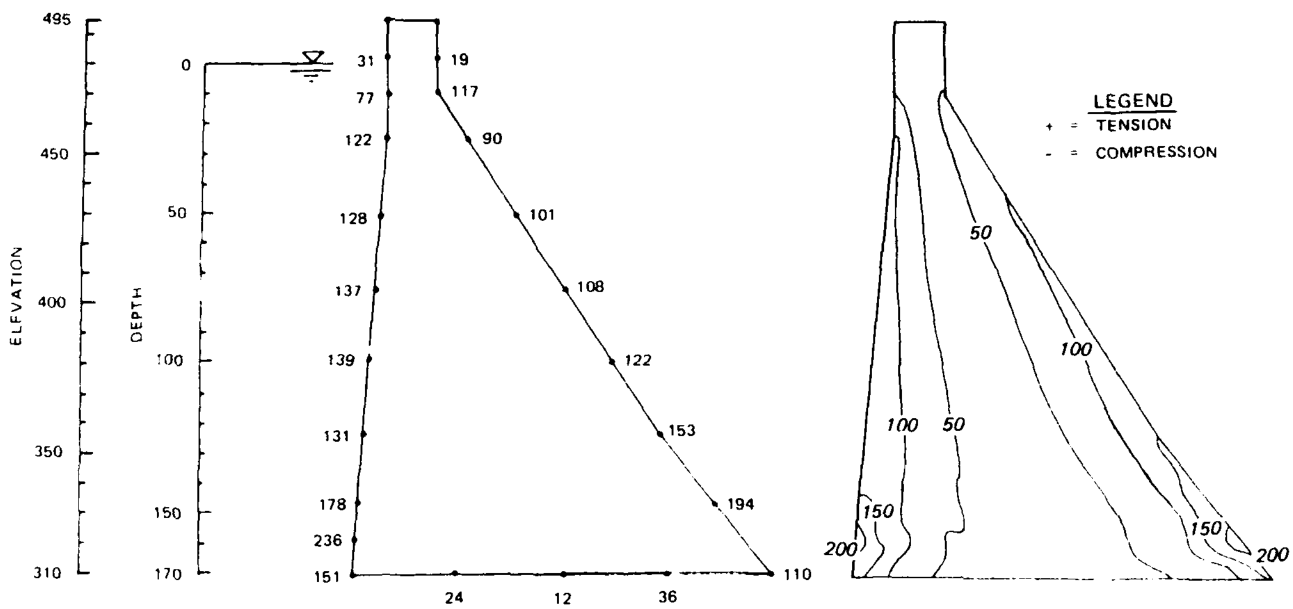


Figure 31. Dynamic stresses (SYY) for Er/Ec ratio of 0.05

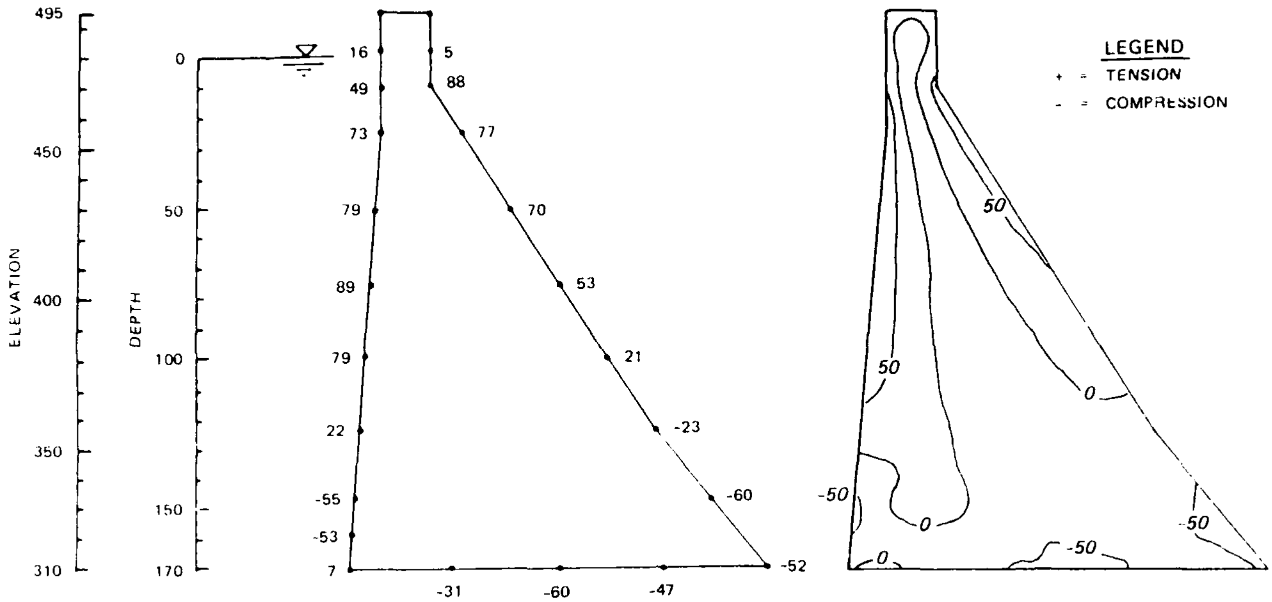


Figure 32. Maximum tensile (static plus dynamic) stresses (SYY) for Er/Ec ratio of 0.05

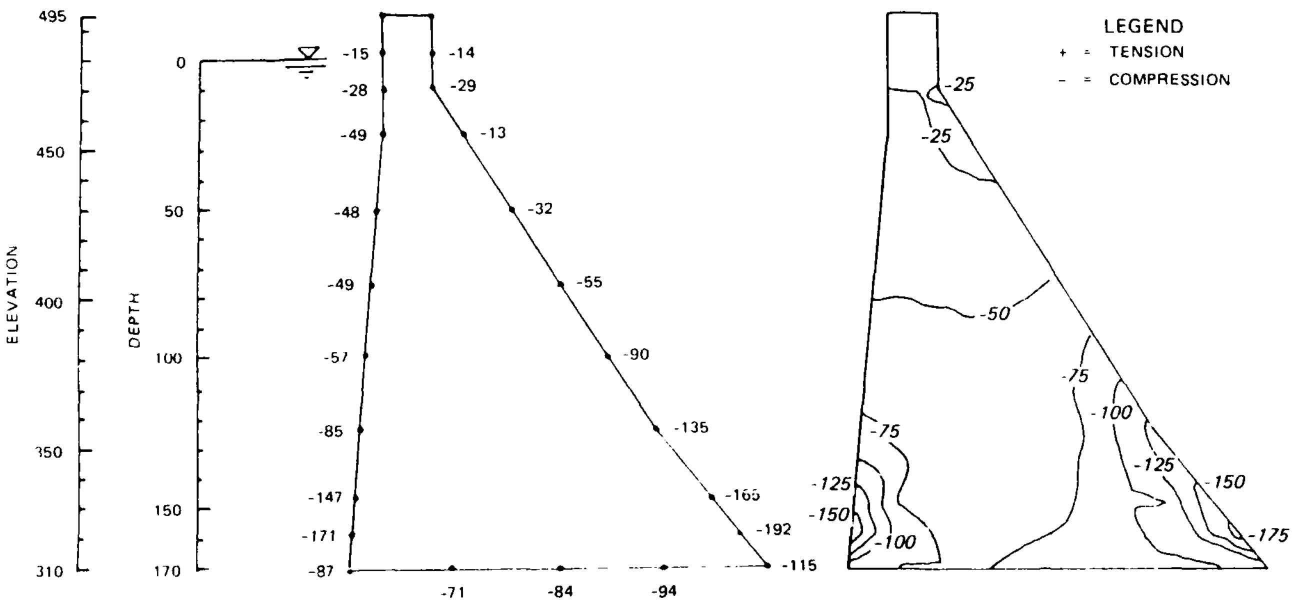


Figure 33. Static stresses (SYY) for Er/Ec ratio of 0.25

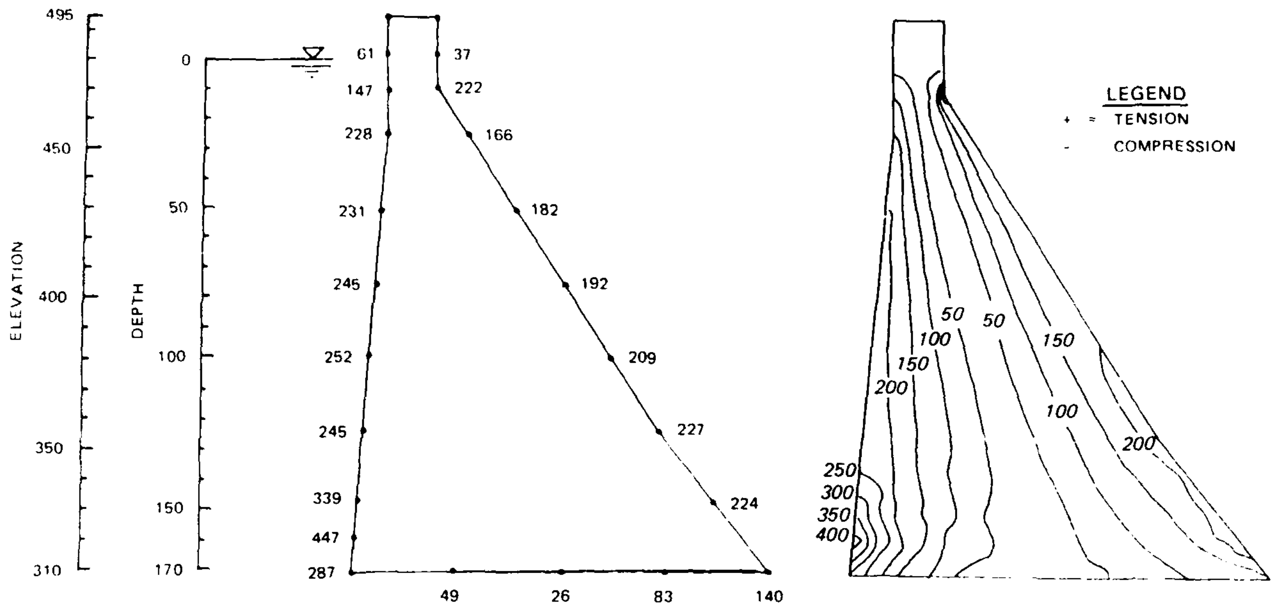


Figure 34. Dynamic stresses (SYY) for Er/Ec ratio of 0.25

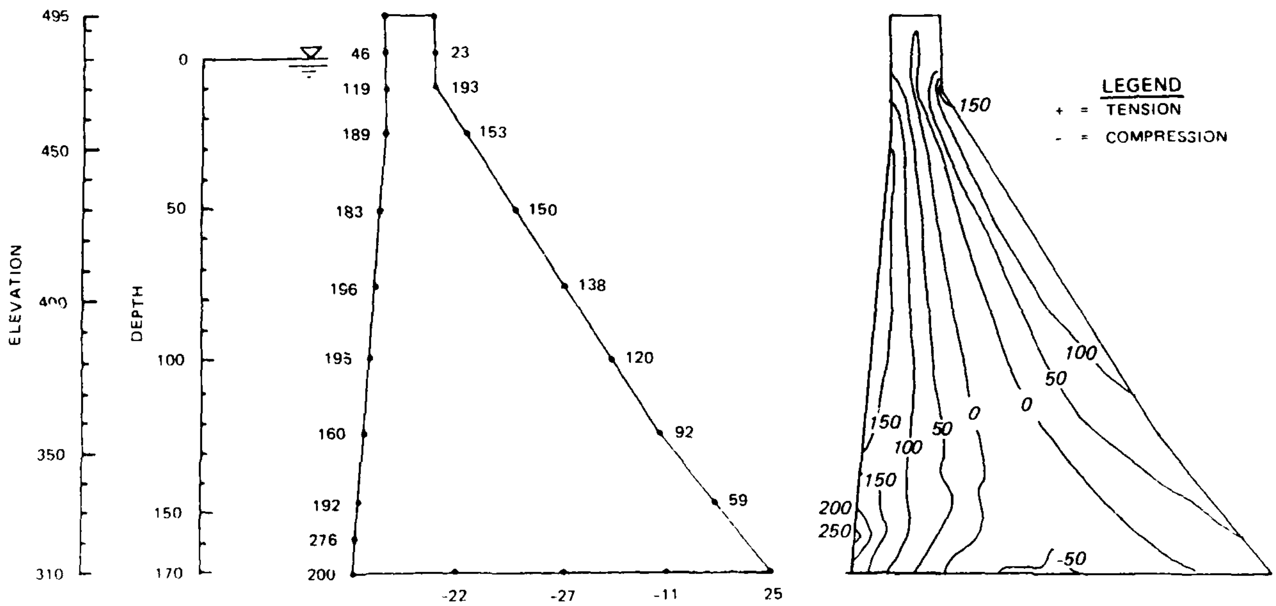


Figure 35. Maximum tensile (static plus dynamic) stresses (SYY) for Er/Ec ratio 0.25

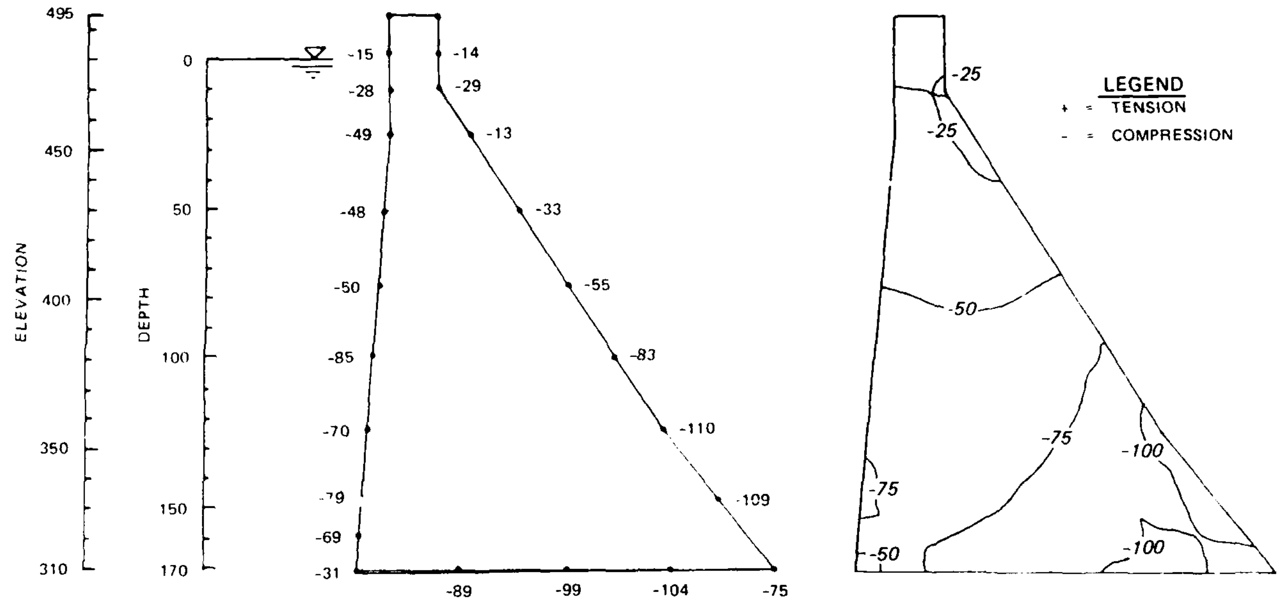


Figure 36. Static stresses (SYY) for Er/Ec ratio of 1.00

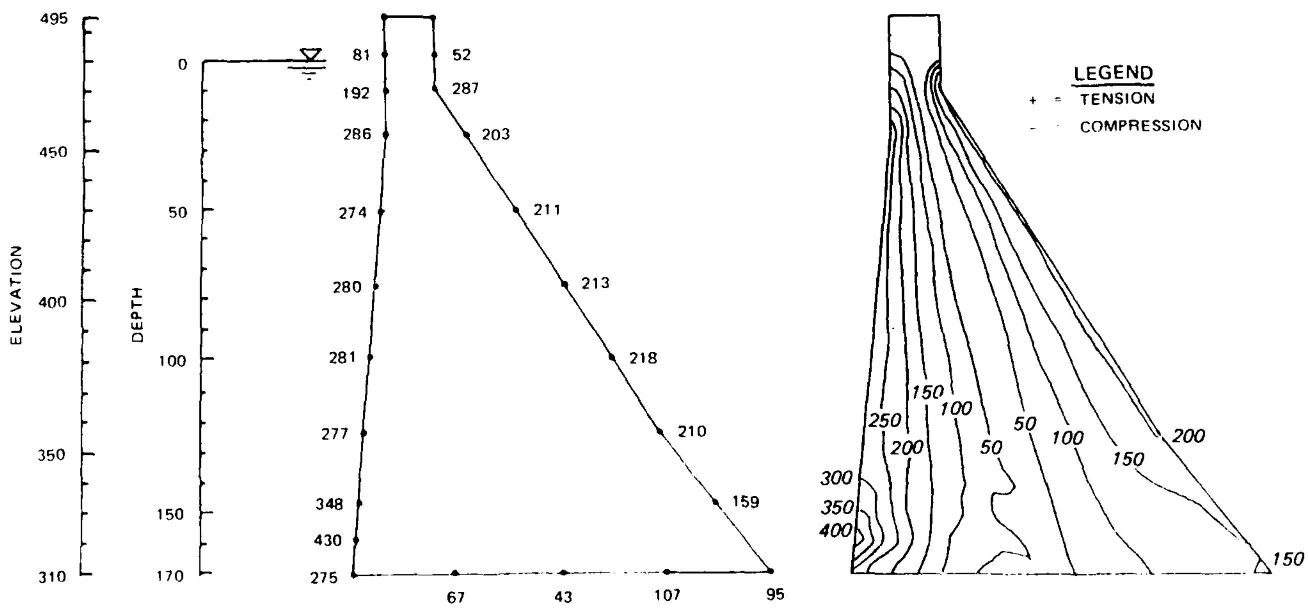


Figure 37. Dynamic stresses (SYY) for Er/Ec ratio of 1.00

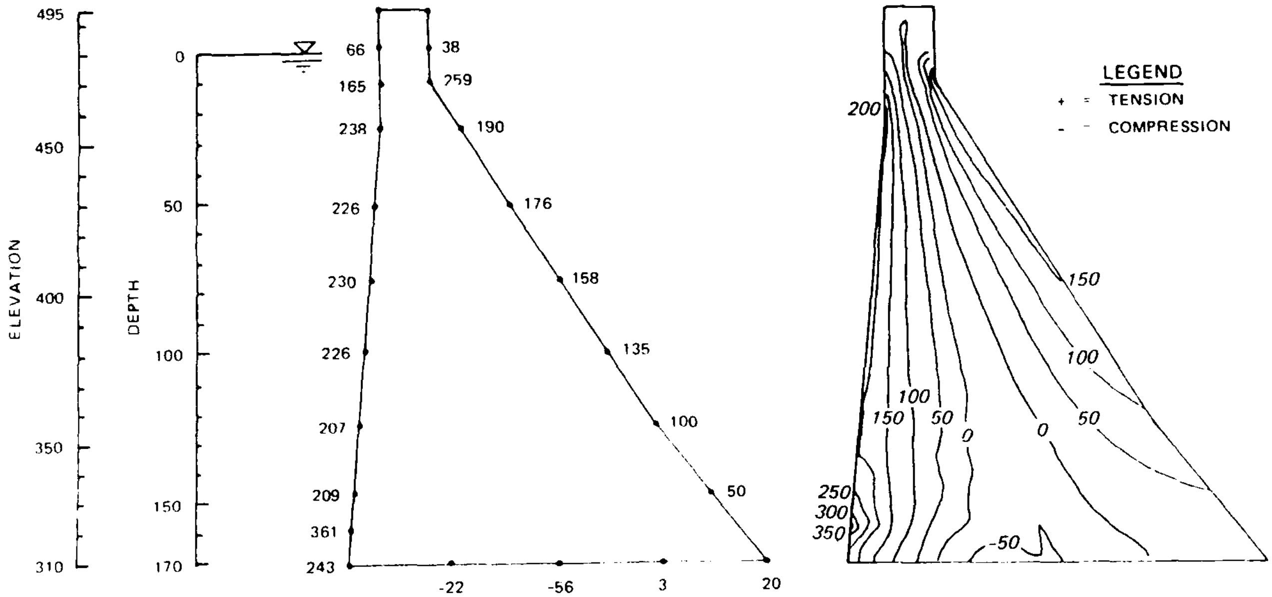


Figure 38. Maximum tensile (static plus dynamic) stresses (SYY) for Er/Ec ratio of 1.00

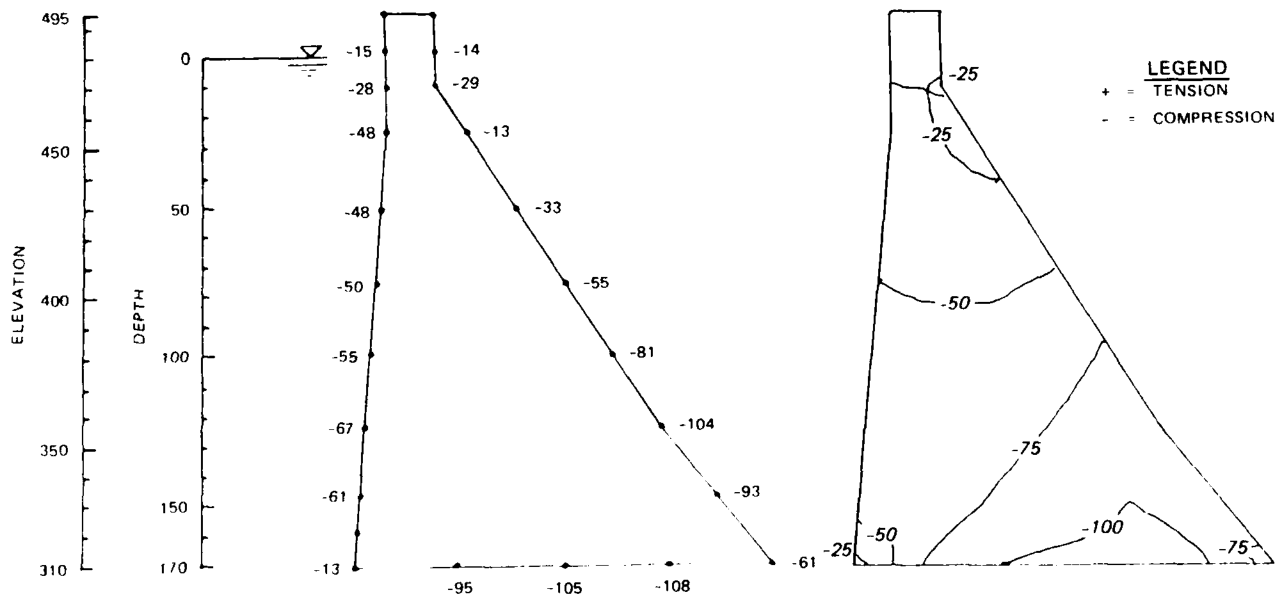


Figure 39. Static stresses (SYY) for Er/Ec ratio of 1.75

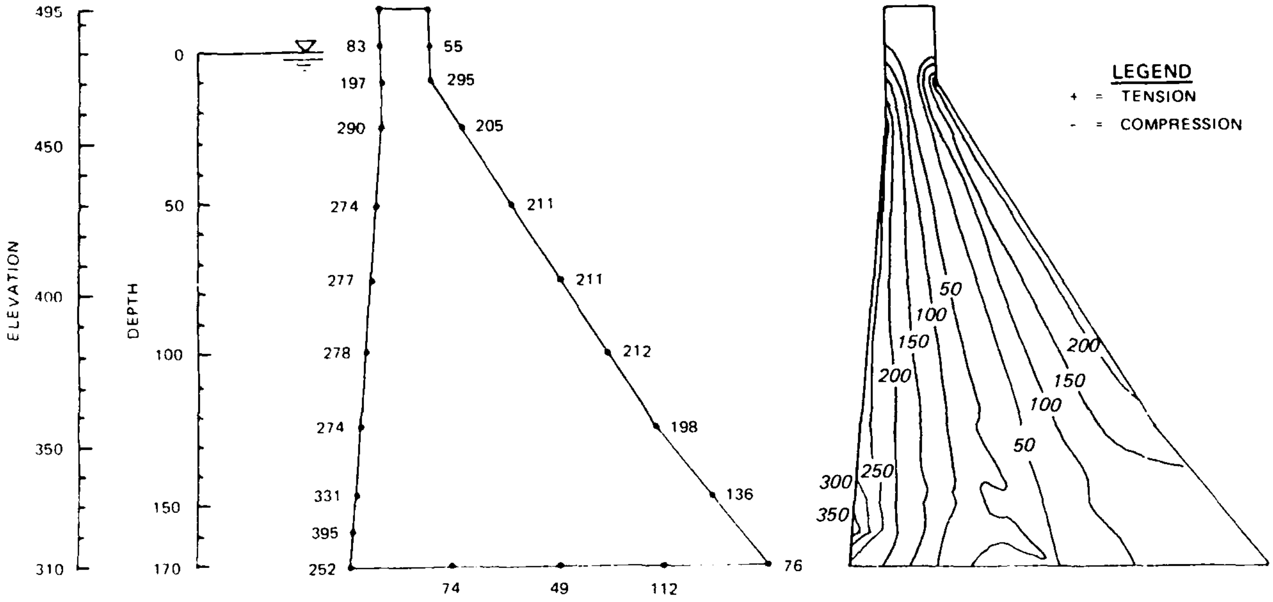


Figure 40. Dynamic stresses (SYY) for Er/Ec ratio of 1.75

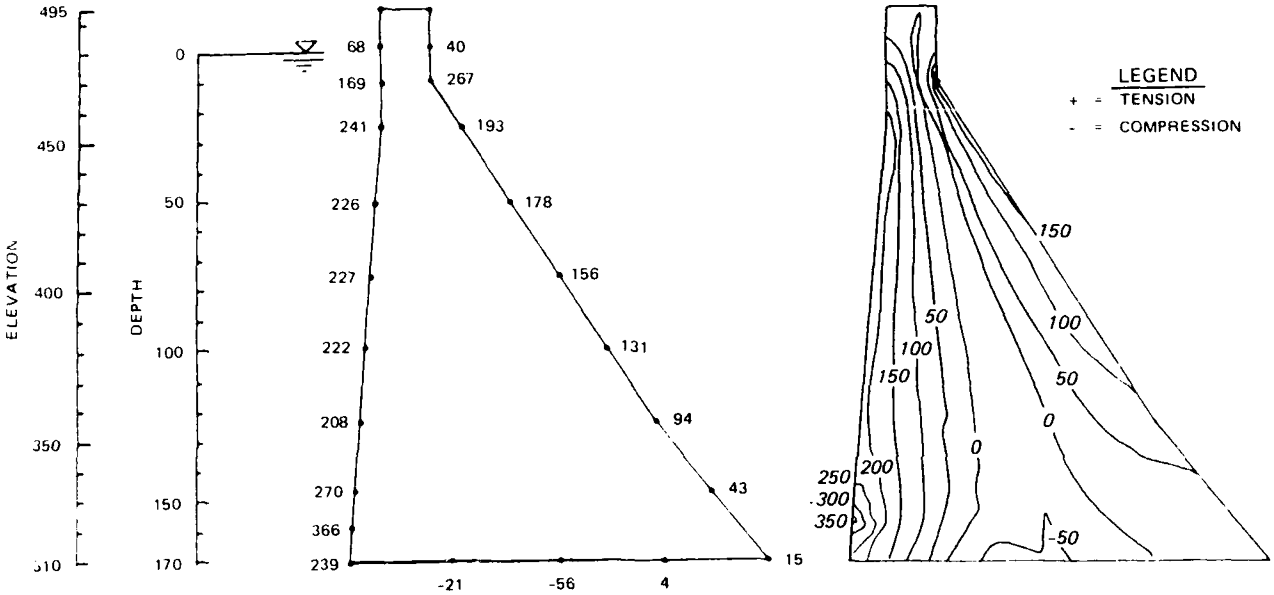


Figure 41. Maximum tensile (static plus dynamic) stresses (SYY) for Er/Ec ratio of 1.75

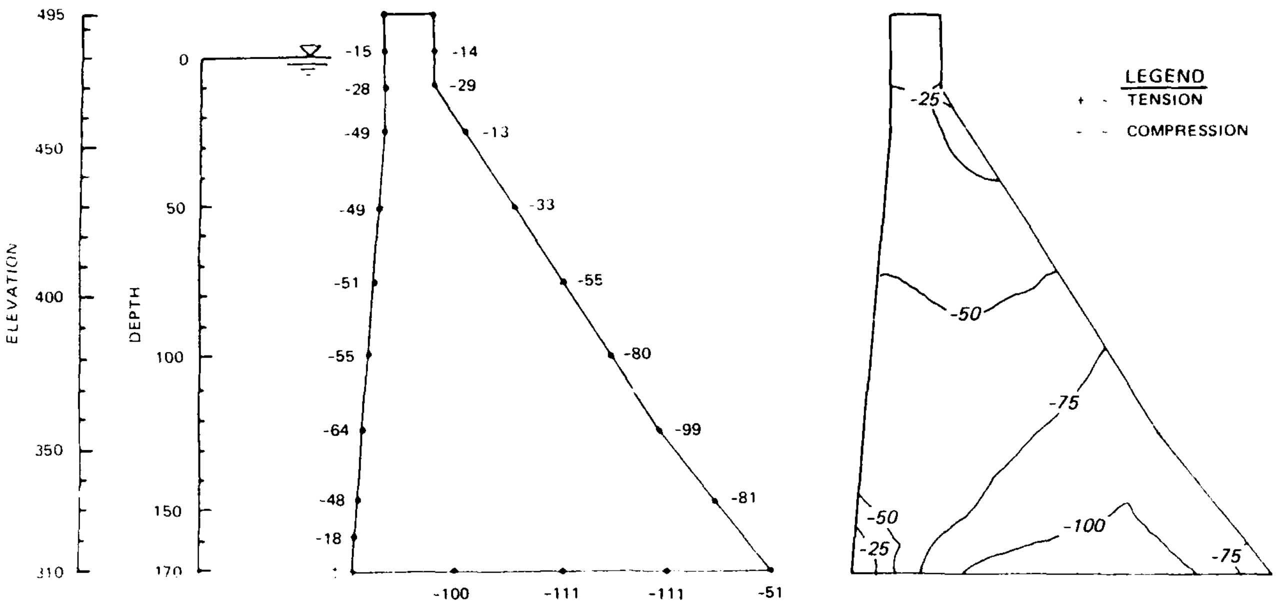


Figure 42. Static stresses (SYY) for Er/Ec ratio of 3.00

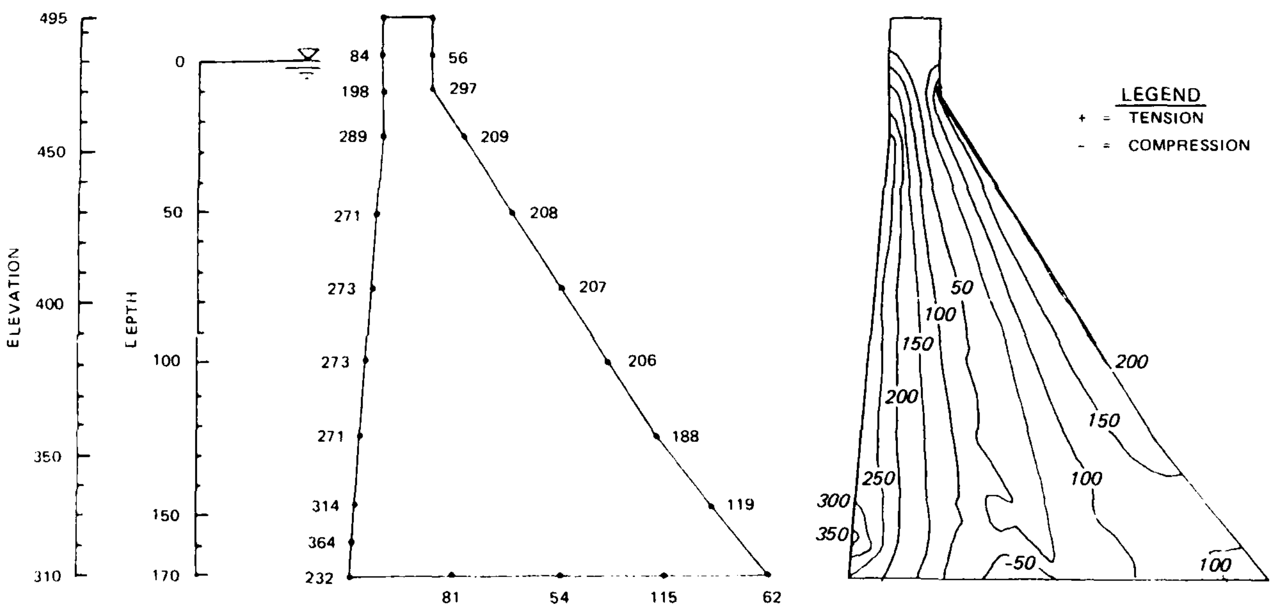


Figure 43. Dynamic stresses (SYY) for Er/Ec ratio of 3.00

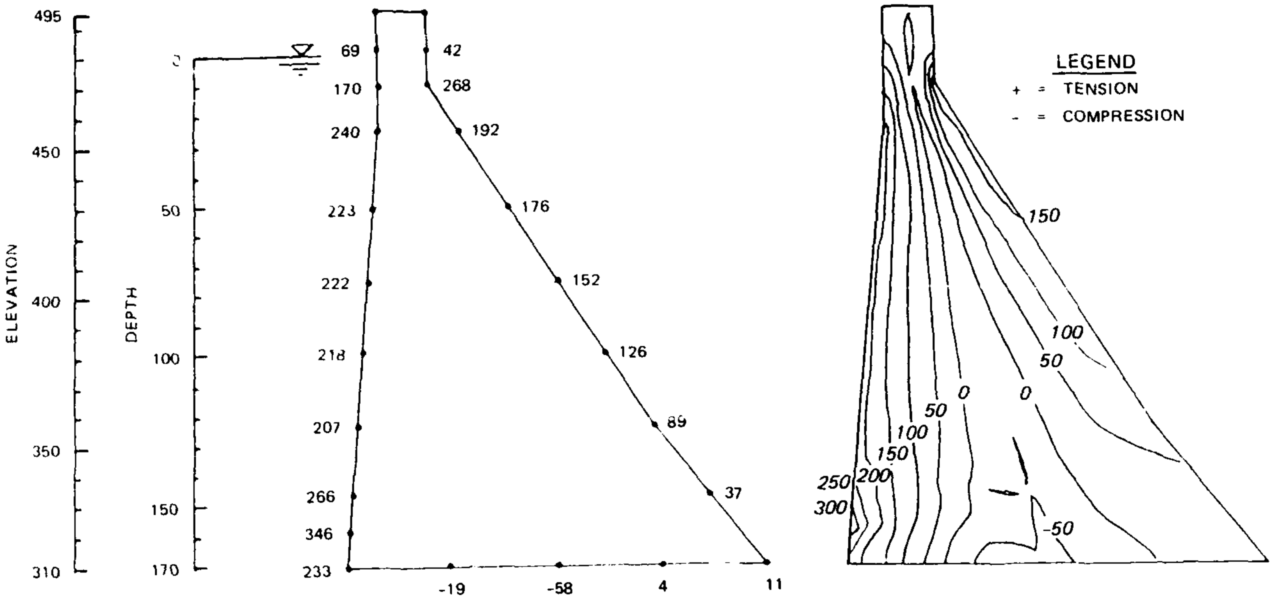


Figure 44. Maximum tensile (static plus dynamic) stresses (SYY) for Er/Ec ratio of 3.00

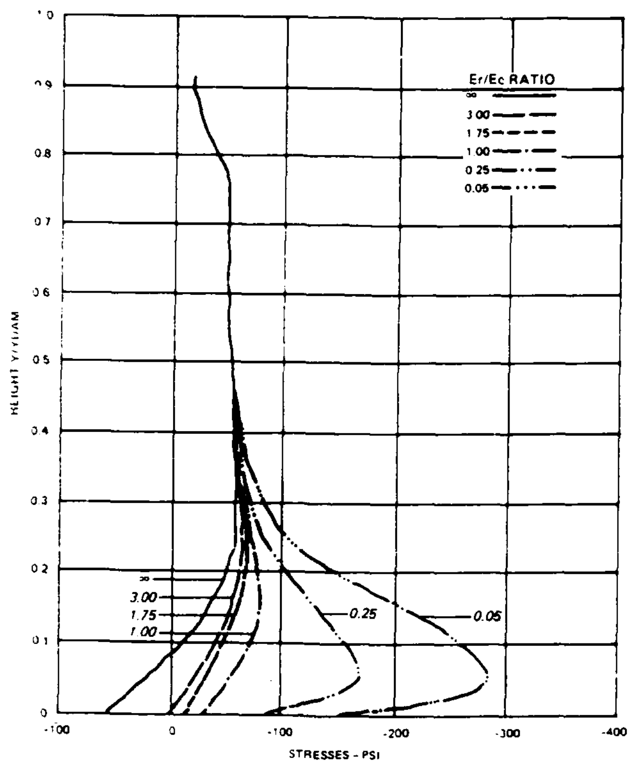


Figure 45. Upstream face: Static stresses (SYY)

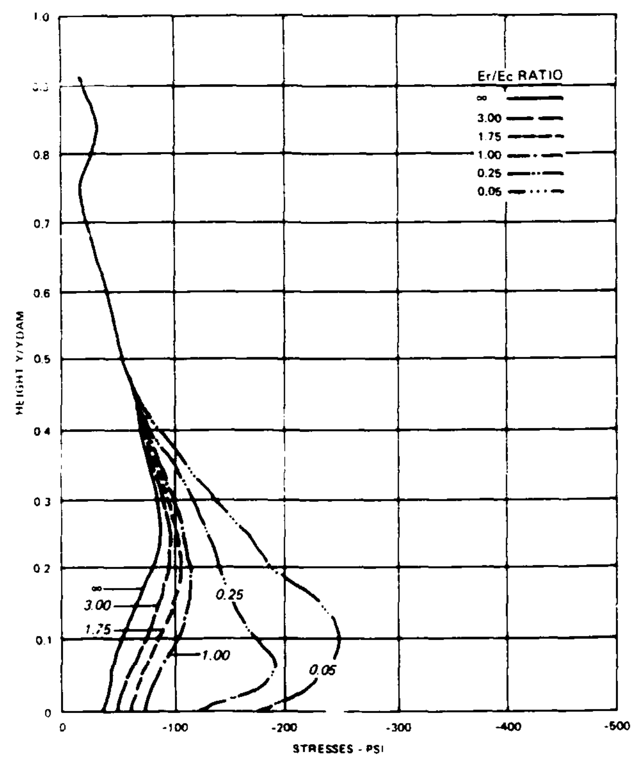


Figure 46. Downstream face: Static stresses (SYY)

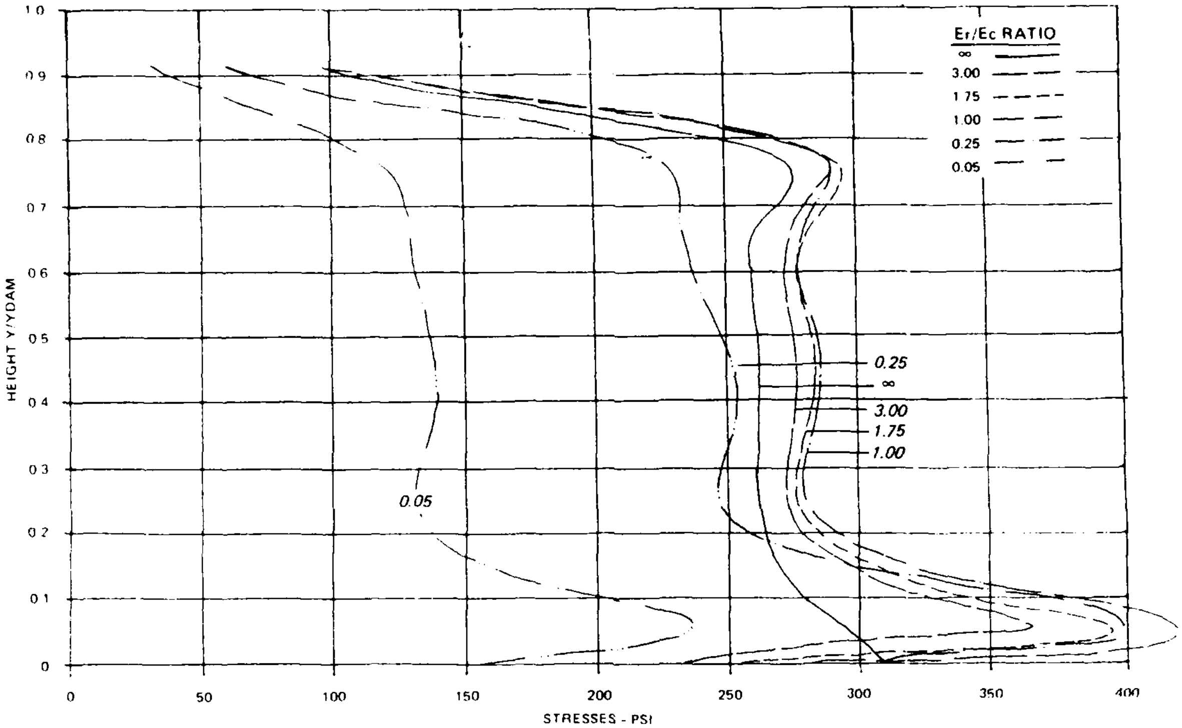


Figure 47. Upstream face: Dynamic tensile stresses (SYY)

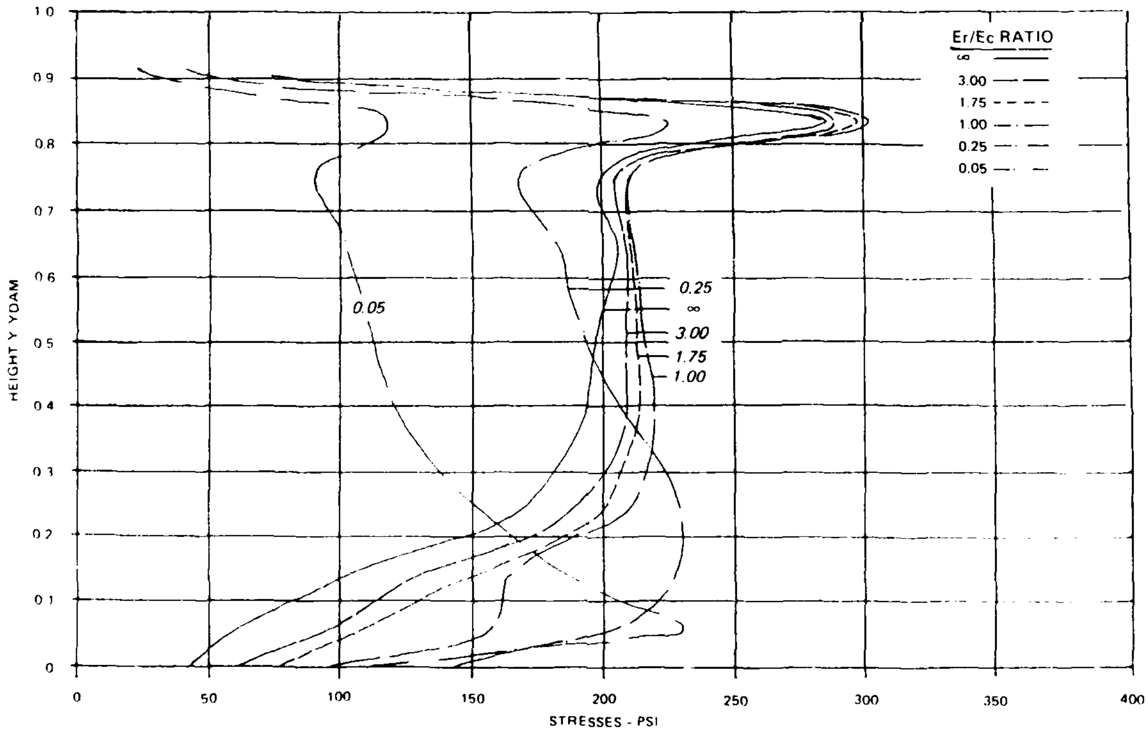


Figure 48. Downstream face: Dynamic tensile stresses (SYY)

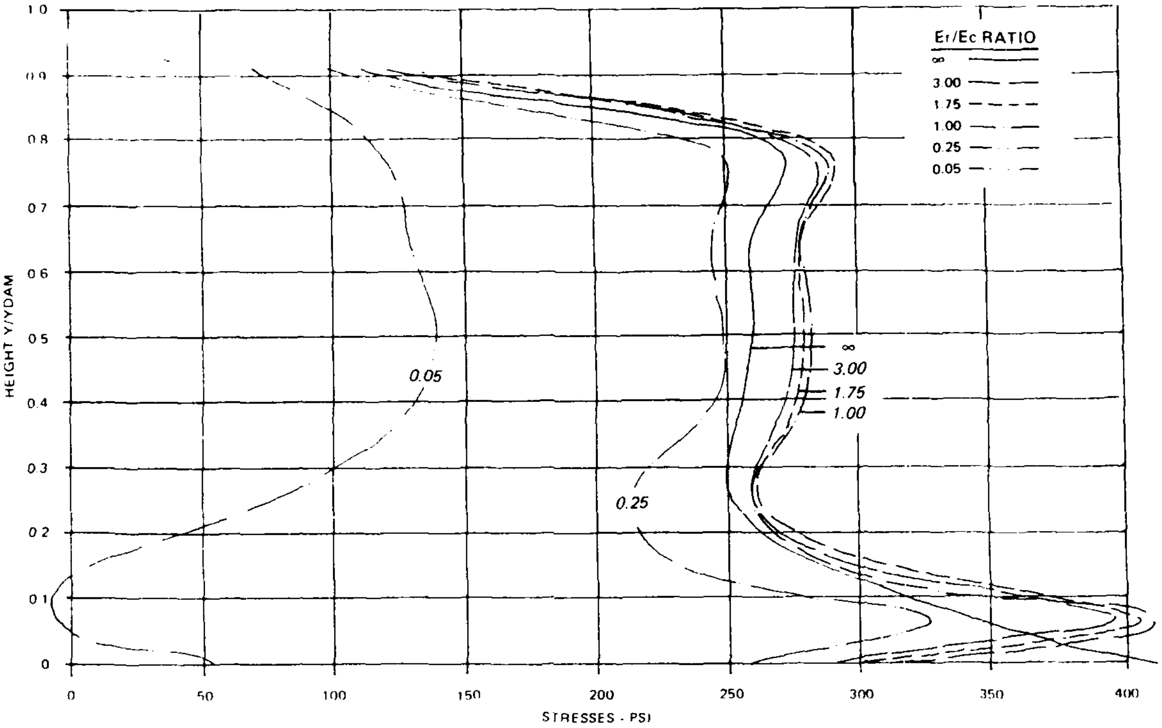


Figure 49. Upstream face: Static + dynamic tensile stresses (SYY)

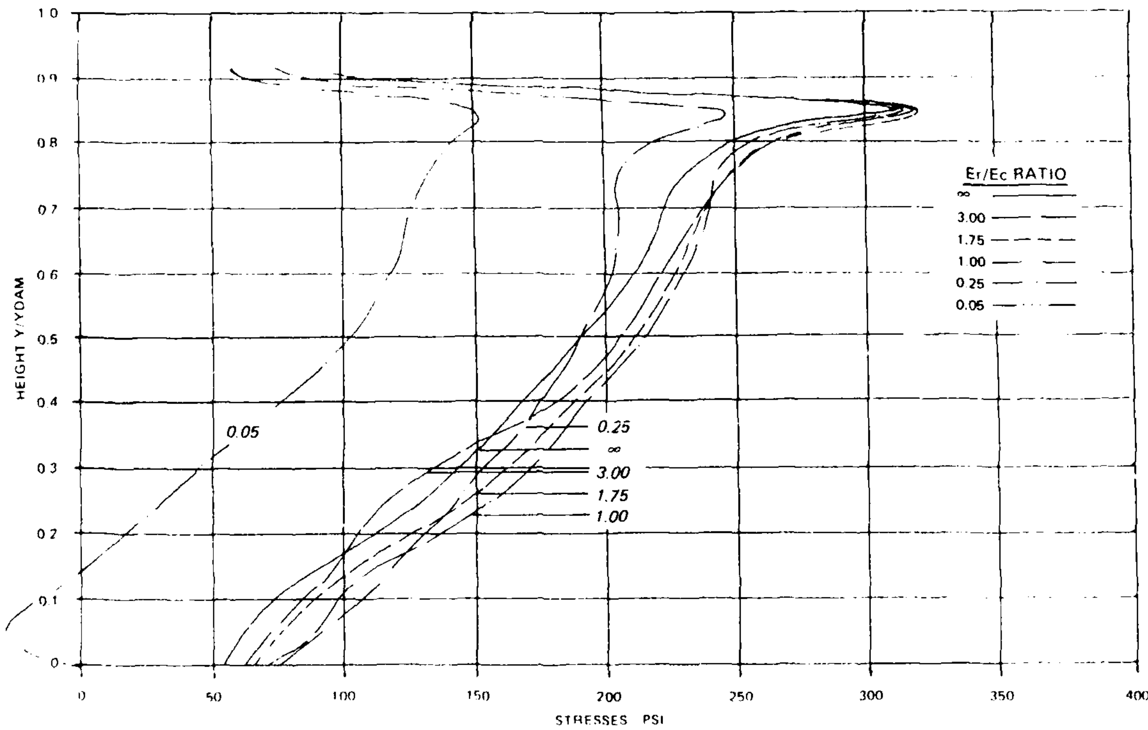


Figure 50. Downstream face: Static + dynamic tensile stresses (SYY)

elasticity decreases, the static stresses (SYY) below midheight of the dam increase. This was confirmed in the Phase Ib report (in preparation).

62. Figures 47 and 48, showing the dynamic stresses, are not quite clear cut. To understand the results, one must first look at the response spectrum. Principally, the first mode of vibration results in the largest participation in the response spectra maximum. Therefore, the model whose fundamental mode shapes period results in the largest corresponding maximum response values will generally have the highest dynamic stresses.

63. Seed's smoothed response spectrum (Figure 5) was used in this analysis. Figure 51 shows the actual digitized response spectrum used for the computer programs input. The first mode periods associated with the various Er/Ec ratios are plotted on this response curve along with the corresponding maximum response values. It so happens, in this example, that the model with an Er/Ec of infinite results in the peak maximum response of 0.63g. As the Er/Ec ratio decreases, so does the response acceleration. This would indicate

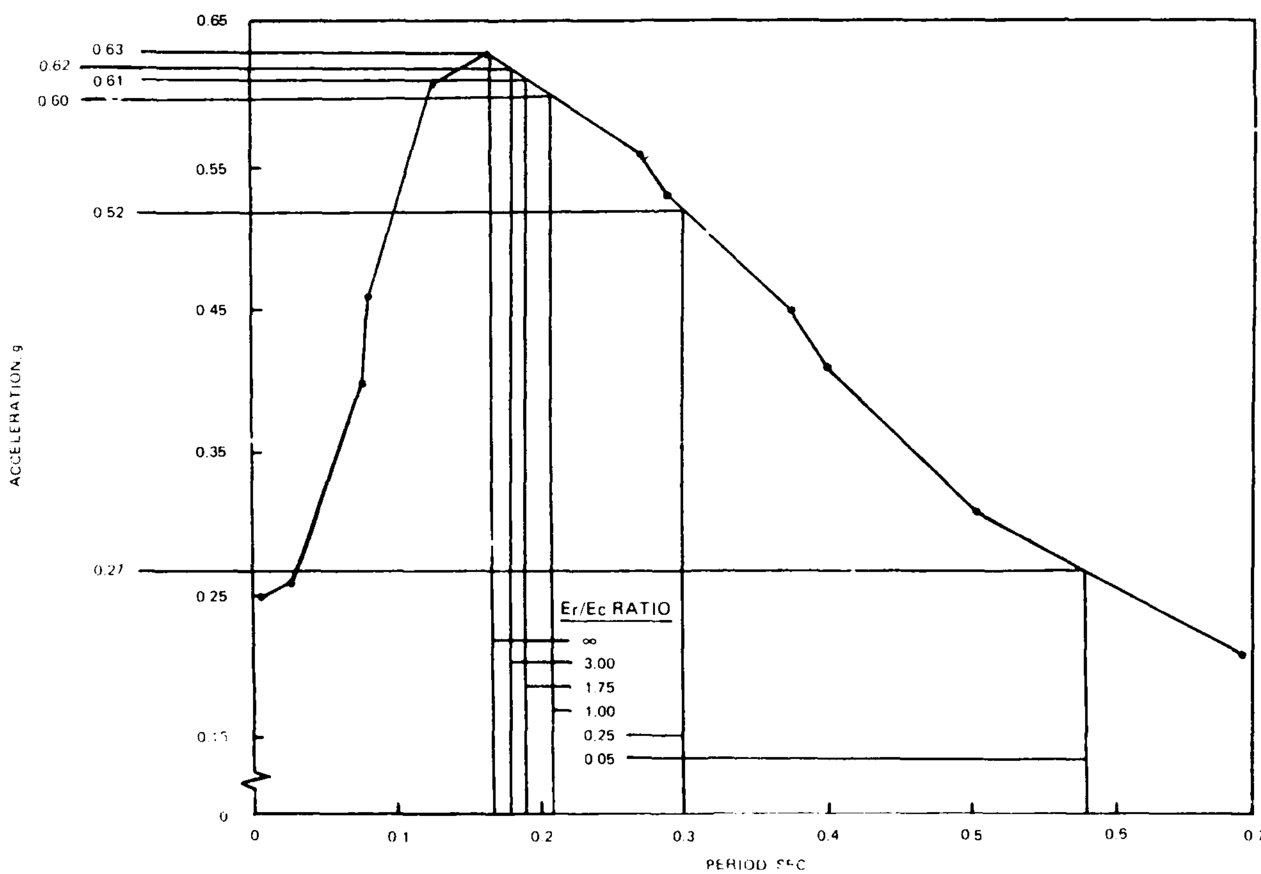


Figure 51. Digitized response spectrum

that the analysis using the Er/Ec ratio of infinite would result in the highest dynamic stresses, and stresses would decrease as the Er/Ec decreased.

64. In this particular case, results shown in Figures 47 and 48 do not quite bear this out. The maximum stresses resulted from an Er/Ec ratio of 1.00. Er/Ec ratios of 1.75, 3.00, and infinite resulted in slightly decreasing stresses. This is opposite of what would be expected and must be attributed to foundation-structure interaction. Stresses for the four highest Er/Ec ratios were, however, generally within 5 percent of each other.

65. Er/Ec ratios of 0.25 and 0.05 did result in lower stresses, as expected, except in the lower portion of the dam.

66. Figures 49 and 50 show the combined static and dynamic stresses. Comparison of the various plots show that combined stresses for Er/Ec ratios above 0.25 are within 25 percent of each other except at the toe of the dam. This indicates, for this particular case, the finite element grid need not include a foundation until the foundation materials modulus of elasticity is less than 25 percent of that in the dam.

PART V: SUMMARY AND RECOMMENDATIONS

Summary

67. The primary objective of this study was to illustrate an approach for performing a finite element response-spectrum dynamic analysis of a gravity dam. The illustrations should serve the beginning finite element analyst with a better understanding of the behavior of a gravity dam which is subject to dynamic loading.

68. The analysis of a monolith similar to those of the Richard B. Russell Dam determined that a mesh with four elements across the base was a reasonable compromise between accuracy and cost in the dynamic analysis. The finite element grid need not include a foundation block as long as the foundation materials modulus of elasticity is at least 25 percent of that in the dam.

69. Results of the comparison between a FEM analysis using Westergaard's (1933) "added mass" and Chopra's (1978) "simplified response spectrum" method showed the simplified method to be conservative, but not excessively so. Results indicate the simplified method could be used to make a first-cut estimate of the surface stresses.

Recommendations

70. Conclusions reached in this report were based on studies using a single monolith size. Varying the overall dimensions of the dams may significantly alter the results. This report should illustrate to the engineer the importance of making verification studies to ensure the use of a proper mesh size and the necessity of including a foundation from which usable results can be obtained. The analyst should develop and analyze finite element models of simplified structures, then extrapolate the information gained to the modeling of the real structure.

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APPENDIX A: MODELING OF HYDRODYNAMIC EFFECTS

1. The "added mass" applied to a structure to simulate the hydrodynamic effects can be computed using Westergaard's formula, EM 1110-2-2200 (Department of the Army 1958),* which gives the hydrodynamic pressure at a depth y below the water surface as:

$$p = C_e \alpha \sqrt{hy} \ \mathrm{lb/ft^2} \tag{1}$$

$$P = \frac{2}{3} C_e \frac{a}{g} \ y\sqrt{hy} \tag{2}$$

where

- p = hydrodynamic pressure at depth y below water surface, pounds per square foot
- P = total pressure to depth y from surface using the parabolic, pounds per square foot approximation
- h = total depth of water, feet
- α = ratio of earthquake acceleration, a to g
- a = acceleration due to the earthquake, feet per second squared
- g = gravitation acceleration, 32.2 $\mathrm{ft/sec^2}$
- C_e = a factor depending principally on height of dam and the earthquake vibration period, t_e , sec

2. Westergaard's approximate equation for C_e , which is sufficiently accurate for all usual conditions, in pounds per cubic foot is:

$$C_e = \frac{51}{\sqrt{1 - 0.72\left(\frac{h}{1{,}000t_e}\right)^2}} \ \mathrm{lb/ft^3} \tag{3}$$

Period of vibration, t_e , is usually assumed as 1 sec. The mass per unit area to be added to the face of the dam is then calculated by dividing the pressure by the acceleration, a . This gives:

$$m = \frac{C_e}{g} \ \sqrt{hy} \ \mathrm{lb\text{-}sec^2/ft^3} \tag{4}$$

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* References cited in this appendix are included in the references at the end of the main text.

$$M = \frac{2}{3}\left(\frac{C_e}{g}\right) y\sqrt{hy} \ \text{lb-sec}^2/\text{ft}^2 \qquad (5)$$

where

m = mass per unit area to be added to the face of the dam

M = total mass to depth y

The total mass to be added to a particular area on the face of the dam is then found by integrating this quantity over the area under consideration. These added masses are lumped at the node points of the finite element grid on the face of the dam. This gives:

$$M_i = \frac{2}{3} \frac{C_e}{g} h^{0.5} \left(y_2^{1.5} - y_1^{1.5}\right) \ \text{slugs/ft} \qquad (6)$$

where

M_i = added mass to be applied at node i

y_2 = pool depth to the midpoint between node i and the node directly below

y_1 = pool depth to the midpoint between node i and the node directly above

APPENDIX B: INPUT FILE FOR MESH 2 MODEL

```
STRUDL 'MESH 2' 'FINE MESH'
$ 4 BY 10 MESH
UNITS KIPS FEET
GEN 9 JOI ID 1 1 X 0.0 10.0 Y 0.0
MOD 10 ID 14 Y 18.5
GEN 5 JOI ID 10 1 X 0.0 20.0 Y 9.25
MOD 9 ID 14 Y 18.5
TYPE PLANE STRESS
GEN 10 ELE ID 1 4 FROM 1,14 TO 3 TO 17 TO 15 TO 2 TO 11 TO 16 TO 10
GEN 10 ELE ID 2 4 FROM 3,14 TO 5 TO 19 TO 17 TO 4 TO 12 TO 18 TO 11
GEN 10 ELE ID 3 4 FROM 5,14 TO 7 TO 21 TO 19 TO 6 TO 13 TO 20 TO 12
GEN 10 ELE ID 4 4 FROM 7,14 TO 9 TO 23 TO 21 TO 8 TO 14 TO 22 TO 13
STAT SUPPORT 1 TO 9
CONSTANTS
E 576000. ALL
POISSON 0.20 ALL
DEN 0.150 ALL
ELEM PROP
1 TO 40 TYPE 'IPQQ' THICK 1.0
DAMPING 0.05 4
$
UNITS KIPS FEET SECONDS CYCLES
STORE RESPONSE SPECTRA ACCELERATION LIN VS PERIOD LIN 'SEED' DUMP
$ ACCELERATION (FT/SEC**2) VS PERIOD (SEC)
DAMPING 0.05 FACTOR 0.25
31.78  .0050  33.42  .0260
51.23  .0745  59.70  .0805
78.99  .1260  81.69  .1635
72.61  .2680  68.68  .2890
57.06  .3755  53.19  .4005
40.22  .5055  37.22  .5415
27.05  .6980  23.92  .7955
17.87 1.0905  16.00 1.1945
 9.98 1.6610   8.82 1.8030
END OF RESPONSE SPECTRUM
$
UNITS KIPS FEET SECONDS
INERTIA OF JOI LUMPED
INERTIA OF JOINTS MASS
$ HYDRODYNAMIC 'ADDED MASS'
1 TRANS X 1.41
10 TRANS X 2.76
15 TRANS X 2.68
24 TRANS X 2.60
29 TRANS X 2.51
```

```
29 TRANS X 2.51
38 TRANS X 2.42
43 TRANS X 2.33
52 TRANS X 2.23
57 TRANS X 2.13
66 TRANS X 2.03
71 TRANS X 1.92
80 TRANS X 1.80
85 TRANS X 1.67
94 TRANS X 1.54
99 TRANS X 1.39
108 TRANS X 1.22
113 TRANS X 1.02
122 TRANS X 0.77
127 TRANS X 0.36
$
UNITS LBS INCHES
DYN LOAD 1 'SEED RESPONSE SPECTRUM'
SUPPORT ACC
TRANSLATION X FILE 'SEED'
END OF DYN LOAD
$
$
EIGENPROBLEM PARAMETERS
SOLVE USING SUBSPACE ITERATION
NUMBER OF MODES 4
PERFORM NO STRUM SEQUENCE CHECK ORTHOGONALITY CHECK
TOLERANCE EIGENVAL 1.E-4
END
$
DYN ANAL MODAL
COMPUTE DYNAMIC DISPLACEMENTS FORCES STRESSES MODAL COMB ALL
PRINT DYN DATA
OUTPUT DECIMAL 4
OUTPUT BY LOADING
OUTPUT FIELD E
LIST DYNAMIC EIGENVE
CREATE PSEUDO STATIC LOADING 2 'CQC OF LOADING 1' AS CQC OF LOADING 1
DELETIONS ; LOAD 1
LIST DISPL STRESSES
CALCULATE AVERAGE STRESSES
SAVE DIRECT 'MESSAV1'
FINISH
END OF FILE
??
```

APPENDIX C: NATURAL FREQUENCY CALCULATIONS OF A CANTILEVER BEAM

Elementary Beam Theory

1. This discussion concerns natural frequency of a cantilever beam based only on the elementary engineering theory of beam bending with no secondary effects (Warburton 1964):*

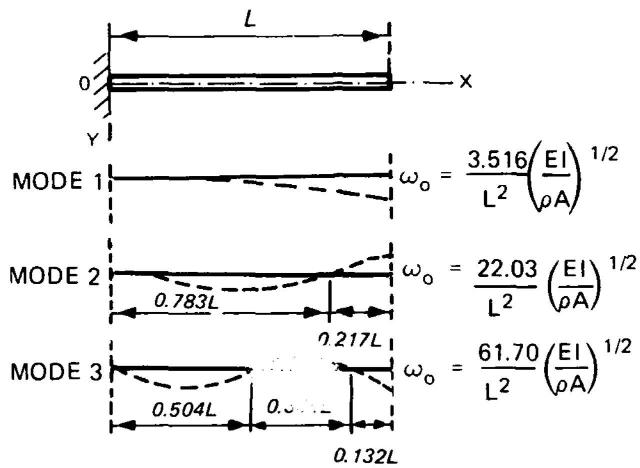

Figure C1. First three modes and frequencies of a uniform cantilever beam based on elementary beam theory (after Warburton 1964)

where

ω_o = natural frequency of beam excluding secondary effects

L = length of cantilever beam (185 ft)

E = modulus of elasticity (576,000 ksf)

I = moment of inertia (42,667 ft^4)

ρ = weight density of material (0.150 k/ft^3)

A = effective total cross-sectional area (80 ft^2)

Mode 1

$$\omega_o = \frac{3.516}{185^2}\left(\frac{576{,}000 \times 42{,}667}{0.15 \times 80}\right)^{1/2} = 4.649 \text{ cyc/sec}$$

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* References cited in this appendix are included in the references at the end of the main text.

Mode 2

$$\omega_o = \frac{22.03}{185^2} \left(\frac{576{,}000 \times 42{,}667}{0.15 \times 80}\right)^{1/2} = 29.13$$

Mode 3

$$\omega_o = \frac{61.70}{185^2} \left(\frac{576{,}000 \times 42{,}667}{0.15 \times 80}\right)^{1/2} = 81.59$$

Timoshenko's Theory

2. This discussion concerns the natural frequency of a cantilever beam including the effect of transverse shear and rotary inertia (Kruszewski 1949). The effect of shear lag and shear deformation of the web is to increase the flexibility of the beam because of the additional deflection that is introduced. The effect of rotary inertia is to increase the dynamic loading on the beam because of the additional inertia loading due to the rotational acceleration of the differential elements of the beam. Considerable lowering of the frequency due to the secondary effects is obtained for the higher-mode numbers, as shown in Figure C2 (Kruszewski 1949)

$$\omega_o = k_{B_o} \sqrt{\frac{EI}{mL^4}} \; ; \; k_s = \frac{1}{L} \sqrt{\frac{EI}{A_s G}}$$

where

ω_o = natural frequency of beam

k_{B_o} = frequency coefficient where shear and rotary inertia are neglected

m = mass of beam per unit length

k_s = coefficient of shear rigidity $\left(\frac{1}{L} \sqrt{\frac{EI}{A_s G}} = 0.21\right)$

A_s = shear area $\left(\frac{5}{6} A_T = 66.7\right)$

μ = Poisson ratio (0.20)

G = shear modulus $\left[\frac{E}{2(1 + \mu)} = 240{,}000 \text{ ksf}\right]$

k_{RI} = coefficient of rotary inertia $\left(\frac{1}{L} \sqrt{\frac{I}{A_T}} = 0.12\right)$

A_T = effective total cross-sectional area (80 ft^2)

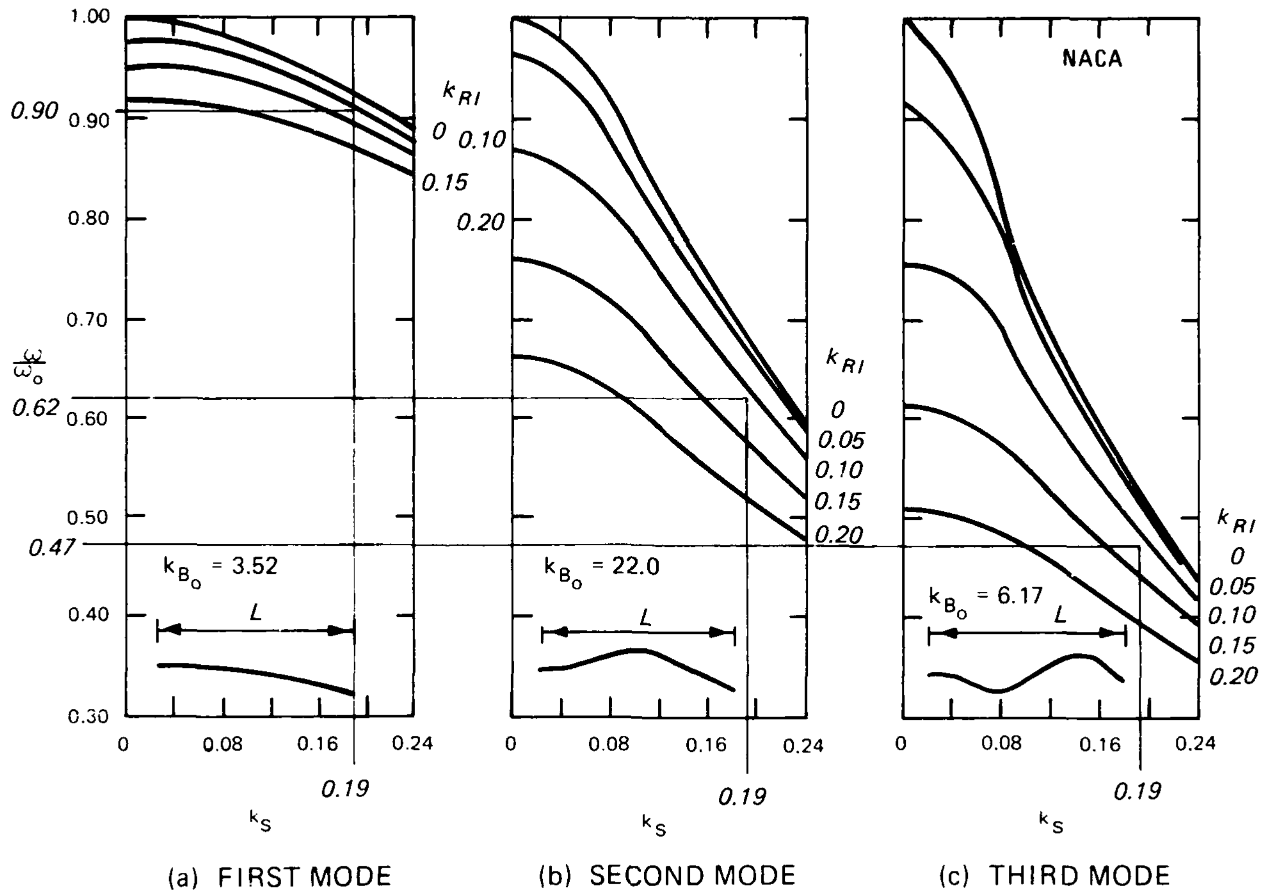

Figure C2. Illustration of how to determine the ratio and natural frequencies of a cantilever beam with and without considering shear and rotary inertia (From Kruszewski 1949)

<u>Mode 1</u>

$$\frac{\omega}{\omega_o} = 0.885 \therefore \omega = 0.885(4.649) = 4.11 \text{ cyc/sec}$$

<u>Mode 2</u>

$$\frac{\omega}{\omega_o} = 0.58 \therefore \omega = 0.58(29.13) = 16.90$$

<u>Mode 3</u>

$$\frac{\omega}{\omega_o} = 0.44 \therefore \omega = 0.44(81.59) = 35.90$$

APPENDIX D: GTSTRUDL INPUT AND OUTPUT FILES FOR GRAVITY DAM EXAMPLE PROBLEM

```
STRUDL 'RBRDAT1' 'RICHARD B. RUSSELL DAM NON-OVERFLOW MONOLITH'
$
UNITS FEET KIPS
$
$ FIRST GENERATE ALL JOINTS AS HAVING ZERO COORDINATES AND THEN
$ GENERATE THE CORNER NODE COORDINATES FOR ALL ELEMENTS IN THE
$ CHANGES MODE. GTSTRUDL WILL ASSUME THAT THE MIDSIDE NODES
$ HAVE COORDINATES OF ZERO.
$
GENE 135 JOI ID 1 1 X 0 0
CHANGES
JOINT COORDINATES
1
9 143.25
29 3.84 46.
37 104.92 46.
85 11.92 143.
93 40.25 143.
99 11.92 160.
107 28.92 160.
113 11.92 170.
121 28.92 170.
127 11.92 185.
135 28.92 185.
$
GEN B 1 9 37 29
XD 4 P EQ
YD 2 P EQ
GEN B 29 37 93 85
XD 4 P EQ
YD 4 P EQ
GEN B 85 93 107 99
XD 4 P EQ
YD 1 P EQ
GEN B 99 107 121 113
XD 4 P EQ
YD 1 P EQ
GEN B 113 121 135 127
XD 4 P EQ
YD 1 P EQ
$
ADDITIONS
$
TYPE PLANE STRESS
GENERATE 4 ELEMENTS ID 1 1 F 1 2 T 3 2 T 17 2 T 15 2 T 2 2 T 11 1 T 16 2 T 10 1
REPEAT 8 ID 4 F 14
STAT SUPPORT 1 TO 9
CONSTANTS
E 576000. ALL
POISSON 0.20 ALL
DEN 0.150 ALL
ELEM PROP
1 TO 36 TYPE 'IPQQ' THICK 1.0
DAMPING 0.05 4
$
UNITS KIPS FEET SECONDS CYCLES
```

```
STORE RESPONSE SPECTRA ACCELERATION LIN VS PERIOD LIN 'SEED' DUMP
$ ACCELERATION (FT/SEC**2) VS PERIOD (SEC)
DAMPING 0.05 FACTOR 0.25
31.78  .0050  33.42  .0260
51.23  .0745  59.70  .0805
78.99  .1260  81.69  .1635
72.61  .2680  68.68  .2899
57.06  .3755  53.19  .4005
40.22  .5055  37.22  .5415
27.05  .6980  23.92  .7955
17.87 1.0905  16.00 1.1945
 9.98 1.6610   8.82 1.8030
END OF RESPONSE SPECTRUM
$
UNITS KIPS FEET SECONDS
INERTIA OF JOI LUMPED
INERTIA OF JOINTS MASS
$ HYDRODYNAMIC 'ADDED MASS'
1 TRANS X 1.55
10 TRANS X 3.02
15 TRANS X 2.91
24 TRANS X 2.79
29 TRANS X 2.74
38 TRANS X 2.68
43 TRANS X 2.53
52 TRANS X 2.37
57 TRANS X 2.20
66 TRANS X 2.01
71 TRANS X 1.81
80 TRANS X 1.58
85 TRANS X 1.14
94 TRANS X .76
99 TRANS X .46
108 TRANS X .23
113 TRANS X .06
$
LOAD 1 'HYDROSTATIC PRESSURES'
ELEMENT LOADS
1 EDGE FOR EDG 4 GLO VAR VX 9.17 9.89 10.61
5 EDGE FOR EDG 4 GLO VAR VX 7.74 8.46 9.17
9 EDGE FOR EDG 4 GLO VAR VX 6.22 6.98 7.74
13 EDGE FOR EDG 4 GLO VAR VX 4.71 5.47 6.22
17 EDGE FOR EDG 4 GLO VAR VX 3.20 3.96 4.71
21 EDGE FOR EDG 4 GLO VAR VX 1.69 2.44 3.2
25 EDGE FOR EDG 4 GLO VAR VX .62 1.15 1.69
29 EDGE FOR EDG 4 GLO VAR VX 0.0 .31 .62
LOAD 2 'DEAD LOAD'
ELEMENT LOADS
1 TO 36 BODY FORCES GLOBAL BY -0.150
UNITS LBS INCHES
STIFFNESS ANALYSIS
DYN LOAD 3 'SEED RESPONSE SPECTRUM'
SUPPORT ACC
TRANSLATION X FILE 'SEED'
END OF DYN LOAD
```

```
END OF DYN LOAD
$
EIGENPROBLEM PARAMETERS
SOLVE USING SUBSPACE ITERATION
NUMBER OF MODES 4
PERFORM NO STURM SEQUENCE CHECK ORTHOGONALITY CHECK
TOLERANCE EIGENVAL 1.E-4
END
$
DYN ANAL MODAL
COMPUTE DYNAMIC STRESSES MODAL COMB ALL
PRINT DYN DATA
OUTPUT DECIMAL 4
OUTPUT BY LOADING
OUTPUT FIELD E
CREATE PSEUDO STATIC LOADING 4 'CQC OF LOADING 3' AS CQC OF LOADING 3
DELETIONS ; LOAD 3 ; ADDITIONS
LOADING COMBINATION 5 'STATIC LOAD = DEAD LOAD + HYDROSTATIC'
COMBINE 5 1 1.0 2 1.0
LOADING COMBINATION 6 'STATIC + DYNAMIC'
COMBINE 6 5 1.0 4 1.0
LIST STRESSES
CALCULATE AVERAGE STRESSES
SAVE DIRECT 'RBRSAV1'
FINISH
END OF FILE
??
```

```
 ************************
 * EIGEN-SOLUTION CHECKS *
 ************************
1
0
```

MODE	EIGENVALUE ((RAD/SEC)**2)	FREQUENCY (RAD/SEC)	FREQUENCY (CYC/SEC)	PERIOD (SEC/CYC)
1	1.441404D+03	3.796583D+01	6.042449D+00	1.654958D-01
2	7.008182D+03	8.371488D+01	1.332364D+01	7.505458D-02
3	1.745866D+04	1.321312D+02	2.102934D+01	4.755262D-02
4	1.975820D+04	1.405639D+02	2.237143D+01	4.469986D-02

....
......
LOADING - 4   COC OF LOADING 3
....
......

AVERAGE STRESSES

JOINT	SURFACE	NUMBER OF ELEMENTS USED IN AVERAGING	SXX	SYY	SXY
1	MIDDLE	1	.614107E+02	.307054E+03	.586600E+02
2	MIDDLE	1	.375439E+02	.187719E+03	.372123E+02
3	MIDDLE	2	.168741E+02	.843705E+02	.254080E+02
4	MIDDLE	1	.155112E+01	.775559E+01	.274340E+02
5	MIDDLE	2	.124030E+02	.620152E+02	.328286E+02
6	MIDDLE	1	.201911E+02	.100955E+03	.414788E+02
7	MIDDLE	2	.234271E+02	.117135E+03	.505917E+02
8	MIDDLE	1	.189337E+02	.946685E+02	.467579E+02
9	MIDDLE	1	.837527E+01	.418764E+02	.351074E+02
10	MIDDLE	1	.218512E+02	.291199E+03	.405163E+02
11	MIDDLE	2	.131711E+02	.980649E+02	.326184E+02
12	MIDDLE	2	.415072E+01	.485892E+02	.385135E+02
13	MIDDLE	2	.167910E+02	.114301E+03	.549161E+02
14	MIDDLE	1	.345494E+02	.573485E+02	.544919E+02
15	MIDDLE	2	.963768E+01	.273605E+03	.205642E+02
16	MIDDLE	2	.640649E+01	.188348E+03	.281176E+02

JOINT	SURFACE	NUMBER OF ELEMENTS USED IN AVERAGING	SXX	SYY	SXY
17	MIDDLE	4	.113174E+02	.107878E+03	.375923E+02
18	MIDDLE	2	.649256E+01	.277409E+02	.361584E+02
19	MIDDLE	4	.803889E+01	.417614E+02	.398218E+02

20	MIDDLE	[illegible]	[illegible]	[illegible]	[illegible]
21	MIDDLE	4	.158722E+02	.124592E+03	.551805E+02
22	MIDDLE	2	.400549E+02	.128263E+03	.647787E+02
23	MIDDLE	2	.616080E+02	.877960E+02	.692417E+02
24	MIDDLE	1	.128240E+02	.262502E+03	.189204E+02
25	MIDDLE	2	.799032E+01	.110515E+03	.322385E+02
26	MIDDLE	2	.102448E+02	.295110E+02	.422681E+02
27	MIDDLE	2	.230212E+02	.117458E+03	.644700E+02
28	MIDDLE	1	.703480E+02	.115959E+03	.946792E+02
29	MIDDLE	2	.148111E+02	.259458E+03	.182203E+02
30	MIDDLE	2	.143842E+02	.189504E+03	.226110E+02
31	MIDDLE	4	.136593E+02	.116193E+03	.290534E+02
32	MIDDLE	2	.128180E+02	.465867E+02	.353889E+02
33	MIDDLE	4	.147166E+02	.213984E+02	.418500E+02
34	MIDDLE	2	.233183E+02	.732214E+02	.527640E+02
35	MIDDLE	4	.316890E+02	.118503E+03	.640914E+02
36	MIDDLE	2	.567955E+02	.151297E+03	.872080E+02
37	MIDDLE	2	.799739E+02	.166852E+03	.113214E+03
38	MIDDLE	1	.138251E+02	.259909E+03	.199375E+02
39	MIDDLE	2	.158696E+02	.117870E+03	.270551E+02
40	MIDDLE	2	.209927E+02	.167287E+02	.437668E+02
41	MIDDLE	2	.387573E+02	.116226E+03	.787697E+02
42	MIDDLE	1	.204586E+02	.179322E+03	.180295E+03
43	MIDDLE	2	.202504E+02	.259398E+03	.181361E+02
44	MIDDLE	2	.206294E+02	.188344E+03	.195908E+02

45	MIDDLE	4	.204424E+02	.118185E+03	.247839E+02
46	MIDDLE	2	.219758E+02	.524039E+02	.335307E+02
47	MIDDLE	4	.258850E+02	.158281E+02	.431644E+02

48	MIDDLE	2	.353258E+02	.674961E+02	.589414E+02
49	MIDDLE	4	.458815E+02	.116328E+03	.765793E+02
50	MIDDLE	2	.630091E+02	.163950E+03	.981830E+02
51	MIDDLE	2	.790476E+02	.191160E+03	.120945E+03
52	MIDDLE	1	.152345E+02	.258953E+03	.198178E+02
53	MIDDLE	2	.199744E+02	.119400E+03	.249202E+02
54	MIDDLE	2	.298307E+02	.137059E+02	.449655E+02
55	MIDDLE	2	.489173E+02	.112330E+03	.803378E+02
56	MIDDLE	1	.809292E+02	.187708E+03	.125547E+03
57	MIDDLE	2	.230213E+02	.258737E+03	.173743E+02
58	MIDDLE	2	.249376E+02	.188218E+03	.187543E+02
59	MIDDLE	4	.250137E+02	.118647E+03	.235081E+02
60	MIDDLE	2	.269698E+02	.539865E+02	.327331E+02
61	MIDDLE	4	.323242E+02	.154126E+02	.434965E+02
62	MIDDLE	2	.421556E+02	.655774E+02	.607229E+02
63	MIDDLE	4	.517840E+02	.115244E+03	.795188E+02
64	MIDDLE	2	.679643E+02	.157466E+03	.102482E+03
65	MIDDLE	2	.832609E+02	.195828E+03	.125400E+03
66	MIDDLE	1	.158130E+02	.257045E+03	.208188E+02
67	MIDDLE	2	.221247E+02	.119959E+03	.244803E+02
68	MIDDLE	2	.336298E+02	.126982E+02	.458989E+02
69	MIDDLE	2	.528093E+02	.111313E+03	.828141E+02
70	MIDDLE	1	.827361E+02	.189392E+03	.128008E+03
71	MIDDLE	2	.285118E+02	.256652E+03	.172216E+02
72	MIDDLE	2	.300469E+02	.187218E+03	.186547E+02
73	MIDDLE	4	.888176E+02	.118892E+03	.805579E+02

74	MIDDLE	2	.898281E+02	.536191E+02	.300587E+02

75	MIDDLE	4	.343432E+02	.180665E+02	.414738E+02
76	MIDDLE	2	.438913E+02	.674227E+02	.590095E+02
77	MIDDLE	4	.536913E+02	.116801E+03	.785881E+02
78	MIDDLE	2	.682636E+02	.158795E+03	.101559E+03
79	MIDDLE	2	.816413E+02	.197946E+03	.123380E+03
80	MIDDLE	1	.149529E+02	.262256E+03	.202845E+02
81	MIDDLE	2	.205873E+02	.120186E+03	.235227E+02
82	MIDDLE	2	.327298E+02	.117058E+02	.463028E+02
83	MIDDLE	2	.518710E+02	.108631E+03	.841059E+02
84	MIDDLE	1	.831154E+02	.184455E+03	.128747E+03
85	MIDDLE	2	.312545E+02	.273666E+03	.120145E+02
86	MIDDLE	2	.325673E+02	.192617E+03	.107742E+02
87	MIDDLE	4	.294260E+02	.116815E+03	.168661E+02
88	MIDDLE	2	.305901E+02	.497536E+02	.270764E+02
89	MIDDLE	4	.355242E+02	.207250E+02	.382900E+02
90	MIDDLE	2	.445045E+02	.742834E+02	.574614E+02
91	MIDDLE	4	.515714E+02	.128720E+03	.768331E+02
92	MIDDLE	2	.670835E+02	.164229E+03	.982557E+02
93	MIDDLE	2	.842662E+02	.193931E+03	.121566E+03
94	MIDDLE	1	.147312E+02	.242266E+03	.242946E+01
95	MIDDLE	2	.233179E+02	.116398E+03	.224153E+02
96	MIDDLE	2	.392116E+02	.821281E+01	.443562E+02
97	MIDDLE	2	.585982E+02	.100820E+03	.783176E+02
98	MIDDLE	1	.788528E+02	.198396E+03	.127481E+03
99	MIDDLE	2	.273219E+02	.188437E+03	.108211E+02
100	MIDDLE	2	.382185E+02	.147293E+03	.174045E+02
101	MIDDLE	4	.333280E+02	.110257E+03	.336888E+02

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102	MIDDLE	2	.358431E+02	.686228E+02	.415195E+02

103	MIDDLE	4	.440808E+02	.248401E+02	.498775E+02
104	MIDDLE	2	.479702E+02	.275162E+02	.554854E+02
105	MIDDLE	4	.532143E+02	.864327E+02	.608834E+02
106	MIDDLE	2	.484715E+02	.179339E+03	.780168E+02
107	MIDDLE	2	.443542E+02	.284231E+03	.991896E+02
108	MIDDLE	1	.110734E+02	.134791E+03	.241544E+01
109	MIDDLE	2	.177079E+02	.761835E+02	.314950E+02
110	MIDDLE	2	.229505E+02	.138355E+02	.475379E+02
111	MIDDLE	2	.188381E+02	.615378E+02	.424004E+02
112	MIDDLE	1	.751320E+01	.166800E+03	.228270E+02
113	MIDDLE	2	.907028E+01	.799857E+02	.707841E+01
114	MIDDLE	2	.914445E+01	.830036E+02	.203153E+02
115	MIDDLE	4	.101343E+02	.433957E+02	.344623E+02
116	MIDDLE	2	.814697E+01	.224661E+02	.410944E+02
117	MIDDLE	4	.624821E+01	.236211E+01	.469276E+02
118	MIDDLE	2	.165559E+01	.226534E+02	.416144E+02
119	MIDDLE	4	.430172E+01	.479431E+02	.419614E+02
120	MIDDLE	2	.521359E+00	.491463E+02	.224734E+02
121	MIDDLE	2	.399100E+01	.545265E+02	.465581E+01
122	MIDDLE	1	.228097E+00	.348862E+02	.174745E+01
123	MIDDLE	2	.153946E+01	.201508E+02	.152814E+02
124	MIDDLE	2	.272960E+01	.182958E+01	.208576E+02
125	MIDDLE	2	.359986E+01	.158541E+02	.158032E+02
126	MIDDLE	1	.381310E+01	.237499E+02	.433778E+01
127	MIDDLE	1	.749948E+01	.317474E+01	.368088E+01
128	MIDDLE	1	.791986E+01	.297731E+01	.755416E+01
129	MIDDLE	2	.884856E+01	.494117E+01	.121971E+02
130	MIDDLE	1	.541586E+01	.217974E+01	.143868E+02

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131	MIDDLE	2	.260273E+01	.102870E+01	.144684E+02
132	MIDDLE	1	.451574E+01	.236499E+01	.135930E+02
133	MIDDLE	2	.643580E+01	.566196E+01	.250492E+01
134	MIDDLE	1	.333047E+01	.814338E+01	.628055E+01
135	MIDDLE	1	.201818E+01	.814081E+01	.849832E+01

LOADING - 5   STATIC LOAD = DEAD LOAD + HYDROSTATIC

AVERAGE STRESSES

JOINT	SURFACE	NUMBER OF ELEMENTS USED IN AVERAGING	SXX	SYY	SXY
1	MIDDLE	1	.110819E+02	.553094E+02	.779787E+02
2	MIDDLE	1	-.103465E+02	-.517327E+02	.549383E+02
3	MIDDLE	2	-.223563E+02	-.111781E+03	.396302E+02
4	MIDDLE	1	-.249090E+02	-.124545E+03	.393738E+02
5	MIDDLE	2	-.253184E+02	-.126592E+03	.411439E+02
6	MIDDLE	1	-.249529E+02	-.124765E+03	.421484E+02
7	MIDDLE	2	-.229889E+02	-.114945E+03	.430333E+02
8	MIDDLE	1	-.168771E+02	-.843854E+02	.373839E+02
9	MIDDLE	1	-.766458E+01	-.383229E+02	.278253E+02
10	MIDDLE	1	-.325084E+02	.226007E+02	.388895E+02
11	MIDDLE	2	-.257538E+02	-.864170E+02	.433175E+02
12	MIDDLE	2	-.221969E+02	-.113187E+03	.482071E+02
13	MIDDLE	2	-.231782E+02	-.105926E+03	.430868E+02
14	MIDDLE	1	-.252769E+02	-.445954E+02	.384068E+02
15	MIDDLE	2	-.683953E+02	-.238025E+02	.210094E+01
16	MIDDLE	2	-.489602E+02	-.563798E+02	.259240E+02

17	MIDDLE	4	-.274048E+02	-.678694F+02	.402966E+02

18	MIDDLE	2	-.306245E+02	-.901146E+02	.403475E+02
19	MIDDLE	4	-.252892E+02	-.101181E+03	.412445E+02
20	MIDDLE	2	-.263896E+02	-.105635E+03	.411593E+02
21	MIDDLE	4	-.241010E+02	-.103870E+03	.418211E+02
22	MIDDLE	2	-.329969E+02	-.883375E+02	.454555E+02
23	MIDDLE	2	-.413104E+02	-.592511E+02	.473683E+02
24	MIDDLE	1	-.675339E+02	-.527203E+02	-.530788E+01
25	MIDDLE	2	-.392201E+02	-.685369E+02	.332224E+02
26	MIDDLE	2	-.300605E+02	-.870170E+02	.394898E+02
27	MIDDLE	2	-.291154E+02	-.938952E+02	.439190E+02
28	MIDDLE	1	-.436300E+02	-.706800E+02	.560951E+02
29	MIDDLE	2	-.526893E+02	-.603903E+02	-.500335E+01
30	MIDDLE	2	-.497542E+02	-.539732E+02	.109506E+02
31	MIDDLE	4	-.460297E+02	-.593736E+02	.245505E+02
32	MIDDLE	2	-.388519E+02	-.677421E+02	.313501E+02
33	MIDDLE	4	-.346970E+02	-.752517E+02	.363895E+02
34	MIDDLE	2	-.341777E+02	-.809198E+02	.395990E+02
35	MIDDLE	4	-.332449E+02	-.857387E+02	424166E+02
36	MIDDLE	2	-.388976E+02	-.891435E+02	.508758E+02
37	MIDDLE	2	-.441775E+02	-.898754E+02	.820714E+02
38	MIDDLE	1	-.486730E+02	-.562826E+02	-.455522E+01
39	MIDDLE	2	-.447314E+02	-.588417E+02	.185464E+02
40	MIDDLE	2	-.374743E+02	-.684782E+02	.381783E+02
41	MIDDLE	2	-.349645E+02	-.783109E+02	.411574E+02
42	MIDDLE	1	-.398481E+02	-.880162E+02	.572550E+02
43	MIDDLE	2	-.446430E+02	-.581638E+02	-.437357E+0

44	MIDDLE	[illegible]	[illegible]	[illegible]	[illegible]
45	MIDDLE	4	-.412182E+02	-.540718E+02	.137880E+02

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46	MIDDLE	2	-.392995E+02	-.557466E+02	.816078E+02
47	MIDDLE	4	-.373233E+02	-.573478E+02	.285923E+02
48	MIDDLE	2	-.358896E+02	-.618660E+02	.339731E+02
49	MIDDLE	4	-.352434E+02	-.662875E+02	.396251E+02
50	MIDDLE	2	-.343844E+02	-.705607E+02	.449952E+02
51	MIDDLE	2	-.336389E+02	-.789657E+02	.513673E+02
52	MIDDLE	1	-.388331E+02	-.531926E+02	-.448697E+01
53	MIDDLE	2	-.372403E+02	-.514737E+02	.113503E+02
54	MIDDLE	2	-.351778E+02	-.527756E+02	.242215E+02
55	MIDDLE	2	-.326433E+02	-.576530E+02	.342492E+02
56	MIDDLE	1	-.297806E+02	-.673371E+02	.444580E+02
57	MIDDLE	2	-.329753E+02	-.514045E+02	-.383457E+01
58	MIDDLE	2	-.327705E+02	-.497965E+02	.291555E+01
59	MIDDLE	4	-.324131E+02	-.482674E+02	.972134E+01
60	MIDDLE	2	-.316908E+02	-.469950E+02	.154522E+02
61	MIDDLE	4	-.311451E+02	-.464046E+02	.209542E+02
62	MIDDLE	2	-.298185E+02	-.470611E+02	.255272E+02
63	MIDDLE	4	-.288131E+02	-.486800E+02	.299122E+02
64	MIDDLE	2	-.269931E+02	-.518080E+02	.340018E+02
65	MIDDLE	2	-.254146E+02	-.561442E+02	.381407E+02
66	MIDDLE	1	-.277833E+02	-.496458E+02	-.413974E+01
67	MIDDLE	2	-.272633E+02	-.453053E+02	.791888E+01
68	MIDDLE	2	-.263938E+02	-.418981E+02	.170812E+02
69	MIDDLE	2	-.241991E+02	-.416064E+02	.239901E+02
70	MIDDLE	1	-.204287E+02	-.456206E+02	.301283E+02
71	MIDDLE	2	-.285393E+02	-.486277E+02	-.348107E+01

72	MIDDLE	2	-.222587E+02	-.455060E+02	.179196E+01
73	MIDILE	4	-.218840E+02	-.483475E+02	.702822E+01
74	MIDILE	2	-.213946E+02	-.392382E+02	.106926E+02

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75	MIDDLE	4	-.209240E+02	-.362307E+02	.144958E+02
76	MIDDLE	2	-.198286E+02	-.342140E+02	.171117E+02
77	MIDDLE	4	-.189857E+02	-.326816E+02	.195893E+02
78	MIDD'E	2	-.170577E+02	-.323848E+02	.213935E+02
79	MIDDLE	2	-.151873E+02	-.330727E+02	.230977E+02
80	MIDDLE	1	-.174466E+02	-.482268E+02	-.384472E+01
81	MIDDLE	2	-.169396E+02	-.398073E+02	.417205E+01
82	MIDDLE	2	-.157529E+02	-.328185E+02	.998025E+01
83	MIDDLE	2	-.139182E+02	-.273490E+02	.138702E+02
84	MIDDLE	1	-.110204E+02	-.246510E+02	.160023E+02
85	MIDDLE	2	-.121015E+02	-.485388E+02	-.175174E+01
86	MIDDLE	2	-.120108E+02	-.423672E+02	.227445E+00
87	MIDDLE	4	-.120156E+02	-.370754E+02	.228493E+01
88	MIDDLE	2	-.111060E+02	-.330209E+02	.437659E+01
89	MIDDLE	4	-.103293E+02	-.292666E+02	.643482E+01
90	MIDDLE	2	-.982371E+01	-.251210E+02	.823815E+01
91	MIDDLE	4	-.861125E+01	-.210399E+02	.980961E+01
92	MIDDLE	2	-.768748E+01	-.169754E+02	.985442E+01
93	MIDDLE	2	-.702713E+01	-.125916E+02	.989303E+01
94	MIDDLE	1	-.80936.E+01	-.388866E+02	-.600255E+00
95	MIDDLE	2	-.808773E+01	-.322728E+02	.585940E+00
96	MIDDLE	2	-.683717E+01	-.275238E+02	.292728E+01
97	MIDDLE	2	-.624567E-01	-.221501E+02	.661291E+01
98	MIDDLE	1	-.585783E+01	-.140350E+02	.962388E+01

99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
100	MIDDLE	2	-.440525E+01	-.271518E+02	-.882160E+00
101	MIDDLE	4	-.443083E+01	-.268159E+02	-.140148E+01
102	MIDDLE	2	-.446132E+01	-.258832E+02	-.782543E+00

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103	MIDDLE	4	-.461746E+01	-.254771E+02	-.820718E+00
104	MIDDLE	2	-.444755E+01	-.251012E+02	.266994E+00
105	MIDDLE	4	-.392642E+01	-.249455E+02	.132878E+01
106	MIDDLE	2	-.371550E+01	-.268877E+02	.424456E+01
107	MIDDLE	2	-.373698E+01	-.286121E+02	.745835E+01
108	MIDDLE	1	-.198928E+01	-.207291E+02	.181135E+00
109	MIDDLE	2	-.209140E+01	-.207070E+02	-.396658E+00
110	MIDDLE	2	-.230963E+01	-.203485E+02	.789940E-01
111	MIDDLE	2	-.161484E+01	-.205230E+02	.637457E+00
112	MIDDLE	1	-.112840E+01	-.237012E+02	.187764E+01
113	MIDDLE	2	-.883095E-01	-.148887E+02	-.223265E+00
114	MIDDLE	2	-.258294E+00	-.155132E+02	-.898637E-01
115	MIDDLE	4	-.417455E+00	-.157799E+02	-.448724E-01
116	MIDDLE	2	-.533202E+00	-.159263E+02	-.593811E-01
117	MIDDLE	4	-.537927E+00	-.160081E+02	.559651E-01
118	MIDDLE	2	-.326196E+00	-.160713E+02	-.112678E-01
119	MIDDLE	4	-.566922E+00	-.164756E+02	.447048E+00
120	MIDDLE	2	-.247740E+00	-.151040E+02	.995997E-01
121	MIDDLE	2	.305323E+00	-.142975E+02	-.834560E+00
122	MIDDLE	1	.758063E-01	-.762271E+01	-.404860E-02
123	MIDDLE	2	.797404E-01	-.784978E+01	.468646E-01
124	MIDDLE	2	.704252E-01	-.791064E+01	.192118E-01
125	MIDDLE	2	.992450E-01	-.771482E+01	.238601E-01
126	MIDDLE	1	.279740E+00	-.736036E+01	-.239395E+00

127	MIDDLE	1	-.162064E+00	-.293739E-01	.647667E-01
128	MIDDLE	1	-.730186E-01	-.338196E-01	.871388E-01
129	MIDDLE	2	-.498748E-01	-.186645E-01	.104628E+00
130	MIDDLE	1	-.868593E-01	-.476153E-01	.451655E-01
131	MIDDLE	2	.399453E-01	.306953E-02	-.670087E-01

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132	MIDDLE	1	-.197612E-01	-.645047E-01	-.379034E-01
133	MIDDLE	2	-.284191E-01	.842171E-01	-.232170E+00
134	MIDDLE	1	-.208821E+00	-.146271E+00	-.540127E-01
135	MIDDLE	1	-.398046E+00	.176242E+00	.360555E+00

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LOADING - 6 STATIC + DYNAMIC

****

AVERAGE STRESSES

JOINT	SURFACE	NUMBER OF ELEMENTS USED IN AVERAGING	SXX	SYY	SXY
1	MIDDLE	1	.724726E+02	.368363E+03	.136639E+03
2	MIDDLE	1	.271973E+02	.135987E+03	.921506E+02
3	MIDDLE	2	-.548218E+01	-.274109E+02	.650382E+02
4	MIDDLE	1	-.233579E+02	-.116789E+03	.668077E+02
5	MIDDLE	2	-.129153E+02	-.645767E+02	.739725E+02
6	MIDDLE	1	-.476182E+01	-.238091E+02	.836272E+02
7	MIDDLE	3	.438174E+00	.219087E+01	.938250E+02
8	MIDDLE	1	.205661E+01	.102830E+02	.841417E+02
9	MIDDLE	1	.710691E+00	.355346E+01	.689327E+02
10	MIDDLE	1	-.106572E+02	.313800E+03	.794058E+02
11	MIDDLE	3	-.185827E+02	.118479E+02	.759369E+02

12	MIDDLE	2	-.180452E+02	-.645781E+02	.807206E+02
13	MIDDLE	2	-.638118E+01	.837485E+01	.979489E+02
14	MIDDLE	1	.987250E+01	.127531E+02	.988987E+02
15	MIDDLE	2	-.587577E+02	.249808E+03	.226652E+02
16	MIDDLE	2	-.406537E+02	.131962E+03	.540415E+02
17	MIDDLE	4	-.160868E+02	.400082E+02	.838889E+02
18	MIDDLE	2	-.241319E+02	-.623737E+02	.764999E+02

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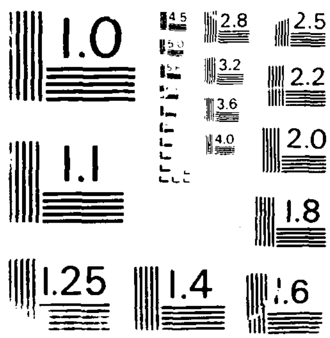
19	MIDDLE	4	-.172504E+02	-.594192E+02	.804663E+02
20	MIDDLE	2	-.144113E+02	-.118820E+02	.873227E+02
21	MIDDLE	4	-.822882E+01	.213814E+02	.969416E+02
22	MIDDLE	2	.706799E+01	.339251E+02	.110234E+03
23	MIDDLE	2	.202976E+02	.285448E+02	.116610E+03
24	MIDDLE	1	-.547100E+02	.209781E+03	.136126E+02
25	MIDDLE	2	-.312297E+02	.419781E+02	.654610E+02
26	MIDDLE	2	-.198157E+02	-.575060E+02	.817559E+02
27	MIDDLE	2	-.609416E+01	.236627E+02	.108389E+03
28	MIDDLE	1	.267181E+02	.453790E+02	.150774E+03
29	MIDDLE	2	-.378782E+02	.199067E+03	.132170E+02
30	MIDDLE	2	-.363700E+02	.136531E+03	.335617E+02
31	MIDDLE	4	-.323704E+02	.568194E+02	.536039E+02
32	MIDDLE	2	-.260339E+02	-.211554E+02	.867390E+02
33	MIDDLE	4	-.199804E+02	-.538533E+02	.782403E+02
34	MIDDLE	2	-.108594E+02	-.769840E+01	.923630E+02
35	MIDDLE	4	-.155591E+01	.327645E+02	.106508E+03
36	MIDDLE	2	.178879E+02	.681537E+02	.138084E+03
37	MIDDLE	2	.357965E+02	.771782E+02	.175285E+03
38	MIDDLE	1	-.348478E+02	.203626E+03	.153823E+02

AD-A212 547 COMPUTER-AIDED STRUCTURAL ENGINEERING (CASE) PROJECT 2/2
THE RESPONSE-SPECTRU (U) ARMY ENGINEER WATERWAYS
EXPERIMENT STATION VICKSBURG MS INFOR P WIERSMA
UNCLASSIFIED AUG 89 WES/TR-ITL-89-6 F/G 13/2 NL


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5

39	MIDDLE	2	-.288618E+02	.610285E+02	.456015E+02
40	MIDDLE	2	-.164816E+02	-.497494E+02	.759452E+02
41	MIDDLE	2	.379275E+01	.389155E+02	.113927E+03
42	MIDDLE	1	.406163E+02	.933054E+02	.177550E+03
43	MIDDLE	2	-.843926E+02	.203235E+03	.137625E+02
44	MIDDLE	2	-.281369E+02	.134877E+03	.242668E+02
45	MIDDLE	4	-.207758E+02	.641131E+02	.385719E+02
46	MIDDLE	2	-.173237E+02	-.334268E+01	.551385E+02

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47	MIDDLE	4	-.114383E+02	-.415197E+02	.717567E+02
48	MIDDLE	2	-.544324E+00	.563008E+01	.929145E+02
49	MIDDLE	4	.106381E+02	.500405E+02	.116204E+03
50	MIDDLE	2	.286847E+02	.833888E+02	.143178E+03
51	MIDDLE	2	.454087E+02	.112194E+03	.172313E+03
52	MIDDLE	1	-.235986E+02	.205761E+03	.153309E+02
53	MIDDLE	2	-.172659E+02	.679859E+02	.362705E+02
54	MIDDLE	2	-.534715E+01	-.390698E+02	.691870E+02
55	MIDDLE	2	.162739E+02	.546774E+02	.114587E+03
56	MIDDLE	1	.511487E+02	.120371E+03	.170005E+03
57	MIDDLE	2	-.995401E+01	.207333E+03	.135397E+02
58	MIDDLE	2	-.783283E+01	.138422E+03	.216698E+02
59	MIDDLE	4	-.739936E+01	.703795E+02	.332294E+02
60	MIDDLE	2	-.472168E+01	.699151E+01	.481853E+02
61	MIDDLE	4	.117901E+01	-.309920E+02	.844507E+02
62	MIDDLE	2	.183378E+02	.185163E+02	.862502E+02
63	MIDDLE	4	.229709E+02	.665648E+02	.109431E+03
64	MIDDLE	2	.409711E+02	.106658E+03	.138484E+03
65	MIDDLE	2	.578464E+02	.139684E+03	.163540E+03
66	MIDDLE	1	-.119703E+02	.207399E+03	.166791E+02

67	MIDDLE	2	-.513867E+01	.746533E+02	.323992E+02
68	MIDDLE	2	.713508E+01	-.891999E+02	.620801E+02
69	MIDDLE	2	.886102E+02	.697064E+02	.106804E+03
70	MIDDLE	1	.623074E+02	.143772E+03	.158134E+03
71	MIDDLE	2	.587190E+01	.208025E+03	.137405E+02
72	MIDDLE	2	.781417E+01	.141712E+03	.184467E+02
73	MIDDLE	4	.695360E+01	.765444E+02	.275841E+02
74	MIDDLE	2	.843343E+01	.143808E+02	.407513E+02
75	MIDDLE	4	.134192E+02	-.201642E+02	.559894E+02

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76	MIDDLE	2	.240626E+02	.332087E+02	.761211E+02
77	MIDDLE	4	.347056E+02	.841194E+02	.981774E+02
78	MIDDLE	2	.512059E+02	.126411E+03	.122952E+03
79	MIDDLE	2	.664539E+02	.164873E+03	.146477E+03
80	MIDDLE	1	-.249374E+01	.214029E+03	.164398E+02
81	MIDDLE	2	.364776E+01	.803790E+02	.276948E+02
82	MIDDLE	2	.169769E+02	-.811127E+02	.562831E+02
83	MIDDLE	2	.379528E+02	.812815E+02	.980661E+02
84	MIDDLE	1	.720950E+02	.159804E+03	.144749E+03
85	MIDDLE	2	.191530E+02	.225127E+03	.102628E+02
86	MIDDLE	2	.205565E+02	.150250E+03	.110017E+02
87	MIDDLE	4	.174104E+02	.797396E+02	.191511E+02
88	MIDDLE	2	.194841E+02	.167327E+02	.314530E+02
89	MIDDLE	4	.251940E+02	-.554165E+01	.447248E+02
90	MIDDLE	2	.348808E+02	.491624E+02	.656896E+02
91	MIDDLE	4	.429602E+02	.107680E+03	.866427E+02
92	MIDDLE	2	.593960E+02	.147253E+03	.108110E+03
93	MIDDLE	2	.772391E+02	.181340E+03	.131459E+03

94	MIDDLE	1	[illegible]	[illegible]	[illegible]
95	MIDDLE	2	.152908E+02	.841856E+02	.830013E+02
96	MIDDLE	2	.323744E+02	-.193112E+02	.472835E+02
97	MIDDLE	2	.523526E+02	.786698E+02	.849305E+02
98	MIDDLE	1	.738956E+02	.184361E+03	.137105E+03
99	MIDDLE	2	.227655E+02	.160588E+03	.109368E+02
100	MIDDLE	2	.278133E+02	.120141E+03	.166124E+02
101	MIDDLE	4	.288971E+02	.834412E+02	.322873E+02
102	MIDDLE	2	.313817E+02	.427396E+02	.407369E+02
103	MIDDLE	4	.394634E+02	-.636959E+00	.428568E+02

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104	MIDDLE	2	.435227E+02	.241506E+01	.557524E+02
105	MIDDLE	4	.498879E+02	.804872E+02	.620122E+02
106	MIDDLE	2	.447580E+02	.152451E+03	.822614E+02
107	MIDDLE	2	.406182E+02	.255619E+03	.106648E+03
108	MIDDLE	1	.908415E+01	.114062E+03	.259658E+01
109	MIDDLE	2	.156165E+02	.554765E+02	.310983E+02
110	MIDDLE	2	.206409E+02	-.651297E+01	.476169E+02
111	MIDDLE	2	.172232E+02	.410148E+02	.430378E+02
112	MIDDLE	1	.638481E+01	.143099E+03	.247046E+02
113	MIDDLE	2	.898197E+01	.650969E+02	.685515E+01
114	MIDDLE	2	.888616E+01	.474904E+02	.202254E+02
115	MIDDLE	4	.971688E+01	.276157E+02	.344174E+02
116	MIDDLE	2	.781377E+01	.853977E+01	.410350E+02
117	MIDDLE	4	.570428E+01	-.136460E+02	.469836E+02
118	MIDDLE	2	.132939E+01	.658213E+01	.416032E+02
119	MIDDLE	4	.373480E+01	.314675E+02	.424084E+02
120	MIDDLE	2	.273819E+00	.340423E+02	.225730E+02
121	MIDDLE	2	.429633E+01	.402890E+02	.382125E+01

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   122      MIDDLE      1      .301902E+00      .273635E+02      .174340E+01
   123      MIDDLE      2      .161920E+01      .183011E+02      .153283E+02
   124      MIDDLE      2      .280003E+01     -.608106E+01      .208769E+02
   125      MIDDLE      2      .369911E+01      .753930E+01      .158271E+02
   126      MIDDLE      1      .389285E+01      .163895E+02      .409839E+01
   127      MIDDLE      1      .734743E+01      .314537E+01      .374564E+01
   128      MIDDLE      1      .784685E+01      .294349E+01      .784130E+01
   129      MIDDLE      2      .879269E+01      .492451E+01      .123017E+02
   130      MIDDLE      1      .532900E+01      .213212E+01      .144319E+02
   131      MIDDLE      2      .264267E+01      .103177E+01      .144005E+02
   132      MIDDLE      1      .449598E+01      .230049E+01      .136551E+02
1                                                                  GTICES V2 M7B   87/04/13. 14.58.33.
 PAGE    72
0
   133      MIDDLE      2      .640738E+01      .568617E+01      .827275E+01
   134      MIDDLE      1      .312165E+01      .199811E+01      .622654E+01
   135      MIDDLE      1      .162014E+01      .831765E+01      .885282E+01
 SAVE DIRECT 'RBRSAV1'
 $$$$ SUBSYSTEM STRUDL ' SAVED IN FILE RBRSAV1  ON GTICES FILE 0
 FINISH
--EOR--
1                                                                  GTICES V2 M7B   87/04/13. 14.58.46.
 PAGE    73
0
        DYNAMIC AREA SUMMARY STATISTICS.

    INITIAL POOL SIZE                          1024
    POOL SIZE INCREMENT                        1024
    NUMBER OF DATA POOL MOVES                   737
    DATA COMPACTIONS                             86
    LOW RELEASED                                  2
    HIGH RELEASED                                 0
    LOW UNRELEASED                                0
    LOW MODULES                                  13
    HIGH MODULES                                  0

    BLOCKS READ FROM DISK                       411
    BLOCKS READ FROM ECS                        963
    SEQUENTIAL READ-AHEADS INITIATED              1
    BLOCKS READ AHEAD DISCARDED                   0
    ONE-PRU READ COUNT                          452
    ONE-PRU WRITE COUNT                         473
    BLOCKS WRITTEN TO DISK                      309
```

APPENDIX E: COLE/CHEEK COMPUTER PROGRAM INPUT AND OUTPUT FILES FOR GRAVITY DAM EXAMPLE PROBLEM

```
*OLD G2DGMI/SDFDAM1,R
*FRN

DO YOU WISH TO SEE INFORMATION FILE ?
  YES (Y) OF NO (CR)
=N
NCOL, B0 (=NROW IF SM =0.), SM ?
=-4,9,0
INPUT DAM GEOMETERY IN FT AND
  ELEVATIONS RELATIVE TO ANY DATUM
ELEV. OF BASE ?
=0.0
SLOPE : RUN TO RISE RATIO
UPSTREAM SLOPE ?
=0.0833
BREAK ELEV. OF UPSTREAM SLOPE ?
=143.
DAM CREST ELEV. ?
=185.
CREST WIDTH ?
=17.
DOWNSTREAM SLOPE ?
=0.67253
BREAK ELEV. OF DOWNSTREAM SLOPE ?
=160.
RADIUS OF TRANSITION ?
=0.
SUFPLY NROW+1 VALUES FOR YH
=0. 23. 46. 70.25 94.5 118.75 143. 160. 170. 185.
```

```
INITIAL PHASE   -  PROGRESS INDICATOR
..........>1/5                                                  *
....................>2/5                                        *
..............................>3/5                              *
........................................>4/5                    *


 MODULUS OF ELASTICITY : E (MILLION PSI) OR F'C (PSI)
=4.
UNIT WEIGHT OF CONCRETE: GAMC (LBF/CF)
=150.
RESERVOIR ELEVATION     : HWATER (FT)
=170.

            NATURAL PERIOD OF DAM =  0.12950 SECONDS

    NATURAL PERIOD OF DAM + WATER =  0.15963 SECONDS ( 6.26 HTZ)


 DO YOU WISH TO USE SEED'S (MEAN)
DESIGN SPECTRUM (5%) FOR ROCK SITES ? (CR)
      SUPPLY YOUR OWN SPECTRUM FILE ? (1)
 SUPPLY SPECTRAL ACCELERATION VALUE ? (2)
(ZERO VALUE FOR STATIC STRESSES ONLY)
=
PEAK GROUND ACCELERATION ?
=0.25


SOLUTION PHASE    -  PROGRESS INDICATOR
..........>1/5                                                  *
....................>2/5                                        *
..............................>3/5                              *
```

NROW = 9 NCOL = 4 NEQ = 90 MBAND = 14 NBLOCK = 1

....................................>4/5 *

1509 HRS., 14 MAY 1987

DAM HEIGHT (FT) : 185.0
POOL HEIGHT (FT) : 170.0
MODULUS (PSI * 10**6) : 4.00
SPECTRAL ACCELERATION (G'S) : 0.61

PRINCIPAL STRESSES AS A FRACTION F'C= 4353.
WHERE E = 33. * (GAMC ** 1.5) * SQRT(F'C)

STRESSES INCLUDE GRAVITY AND HYDROSTATIC LOADS

	UPSTREAM PRINCIPAL STRESSES (%F'C)	DAM HEIGHT (FT)	DOWNSTREAM PRINCIPAL STRESSES (%F'C)
	36. (1%)	177.5	19. (0%)
	122. (3%)	165.0	165. (4%)
	268. (6%)	151.5	295. (7%)
	328. (8%)	130.9	270. (6%)
	329. (8%)	106.6	255. (6%)
	333. (8%)	82.4	237. (5%)
	344. (8%)	58.1	208. (5%)
	379. (9%)	34.5	163. (4%)
>	494. (11%)	11.5	114. (3%)

```
LARGEST PRINCIPLE STRESSES (PSI) AT ELEMENT CENTROID

DOWNSTREAM    333.1       UPSTREAM    224.0


TIME SUMMARIES:
PHASE 1: INITIAL FORMULATION    2.6
PHASE 2: LOAD GENERATION        1.1
PHASE 3: SOLUTION               0.5
PHASE 4: POST PROCESSING        0.2
         TOTAL TIME             4.4



DO YOU WISH TO TRY NEW DAM PROPERITIES ?
 YES (CR) OR NO (N)
*N
(CR) TO STOP, ANY LETTER TO CONTINUE
*
stop
```

[illegible]	[illegible] Computer Program for Design and Analysis [illegible] Retaining Walls and Floodwalls [illegible]	[illegible]
[illegible]	[illegible] Computer Program for Design and Analysis [illegible] (NEWTON)	[illegible]
[illegible]	[illegible] Computer Program for Optimum Nonlinear [illegible] Design of Reinforced Concrete Slabs Under Blast [illegible] (CBARCS)	Mar [illegible]
[illegible]	User's Guide: Computer Program for Design or Investigation of Orthogonal Culverts (CORTCUL)	Mar [illegible]
[illegible] Report K-81-9	User's Guide: Computer Program for Three-Dimensional Analysis of Building Systems (CTABS80)	Aug 1981
Technical Report K-81-2	Theoretical Basis for CTABS80: A Computer Program for Three-Dimensional Analysis of Building Systems	Sep 1981
Instruction Report K-82-6	User's Guide: Computer Program for Analysis of Beam-Column Structures with Nonlinear Supports (CBEAMC)	Jun 1982
Instruction Report K-82-7	User's Guide: Computer Program for Bearing Capacity Analysis of Shallow Foundations (CBEAR)	Jun 1982

(Continued)

[illegible]
[illegible]
[illegible]

WATERWAYS EXPERIMENT STATION REPORTS PUBLISHED UNDER THE COMPUTER-AIDED STRUCTURAL ENGINEERING (CASE) PROJECT

(Continued)

	Title	Date
Technical Report ITL-87-4	Finite Element Studies of a Horizontally Framed Miter Gate Report 5 Alternate Configuration Miter Gate Finite Element Studies—Additional Closed Sections Report 6 Elastic Buckling of Girders in Horizontally Framed Miter Gates Report 7 Application and Summary	Aug 1987
Instruction Report GL-87-1	User's Guide: UTEXAS2 Slope-Stability Package; Volume I, User's Manual	Aug 1987
Instruction Report ITL-87-5	Sliding Stability of Concrete Structures (CSLIDE)	Oct 1987
Instruction Report ITL-87-6	Criteria Specifications for and Validation of a Computer Program for the Design or Investigation of Horizontally Framed Miter Gates (CMITER)	Dec 1987
Technical Report ITL-87-8	Procedure for Static Analysis of Gravity Dams Using the Finite Element Method — Phase Ia	Jan 1988
Instruction Report ITL-88-1	User's Guide: Computer Program for Analysis of Planar Grid Structures (CGRID)	Feb 1988
Technical Report ITL-88-1	Development of Design Formulas for Ribbed Mat Foundations on Expansive Soils	Apr 1988
Technical Report ITL-88-2	User's Guide: Pile Group Graphics Display (CPGG) Post-processor to CPGA Program	Apr 1988
Instruction Report ITL-88-2	User's Guide for Design and Investigation of Horizontally Framed Miter Gates (CMITER)	Jun 1988
Instruction Report ITL-88-4	User's Guide for Revised Computer Program to Calculate Shear, Moment, and Thrust (CSMT)	Sep 1988
Instruction Report GL-87-1	User's Guide: UTEXAS2 Slope-Stability Package; Volume II, Theory	Feb 1989
Technical Report ITL-89-3	User's Guide: Pile Group Analysis (CPGA) Computer Group	Jul 1989
Technical Report ITL-89-4	CBASIN--Structural Design of Saint Anthony Falls Stilling Basins According to Corps of Engineers Criteria for Hydraulic Structures; Computer Program X0098	Aug 1989
Technical Report ITL-89-5	CCHAN--Structural Design of Rectangular Channels According According to Corps of Engineers Criteria for Hydraulic Structures; Computer Program X0097	Aug 1989
Technical Report ITL-89-6	The Response-Spectrum Dynamic Analysis of Gravity Dams Using the Finite Element Method; Phase II	Aug 1989

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